Algebra 1 - Linear & Exponential Functions

Mathematics Vision Project, Secondary Mathematics One, Module 2 (2016)

Review summary

47%

Review completeness

27 of 58 criteria provisionally judged



Gateway 1

Standards and progression

10%

1/10 criteria judged

Gateway 2

Rigor and mathematical practice

83%

10/12 criteria judged

Gateway 3

Teaching and participation

57%

12/21 criteria judged

Gateway 4

Assessment and implementation

27%

4/15 criteria judged

These percentages describe review completeness, not curriculum quality. Quality rating pending.

Key findings

Student tasks connect contextual models, representations, domain decisions and equation forms. Teacher notes provide launch, monitoring and discussion guidance.

The provided standards correlations include incorrect or misplaced codes and need correction before they can serve as a reliable coverage map.

A separate module summative test is not included in the supplied student and teacher PDFs.

Reviewed scope

Module 2, lessons 2.1-2.6; student edition and teacher notes. Provisional findings; unassessed criteria are not zeros. No whole-course rating.

Explore the interactive review

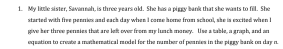
2.1 Connecting the Dots: Piggies and Pools

Explore lesson 2.1

Standards connection

HSF.LE.A.2 asks students to construct linear and exponential functions from relationships. The penny and interest situations require rules linked to tables and graphs, but the variables and representations are supplied.

SE PDF p. 3 (printed p. 1), #1; p. 4 (printed p. 2), #3; TE PDF p. 6, Purpose / Special Note to Teachers


1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day n.

SE PDF p. 3 (printed p. 1), #1

Conceptual understanding

Comparing the penny deposits with the interest account asks students to distinguish additive and multiplicative change. Teacher discussion also contrasts discrete deposits with continuous water flow, connecting a contextual difference to the graph and domain.

SE PDF p. 5 (printed p. 3), #5–7; TE PDF p. 8, Discuss: compare #1 and #2; p. 9, Discuss continuation: domains

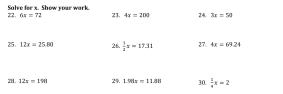
5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

SE PDF p. 5 (printed p. 3), #5

Procedural skill and practice

The GO set practices one-step equations and then asks students to match equations to earlier shopping situations. This maintains supporting equation skills; it does not by itself establish fluency with function construction.

SE PDF p. 7 (printed p. 5), #16–31; pictured #22–30


Solve for x. Show your work.

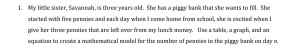
22. $6x = 72$
23. $4x = 200$
24. $3x = 50$
25. $12x = 25.80$
26. $\frac{1}{2}x = 17.31$
27. $4x = 69.24$
28. $12x = 198$
29. $1.98x = 11.88$
30. $\frac{x}{8} = 2$

SE PDF p. 7 (printed p. 5), #22-30

Contextual application

The contexts determine starting amounts, additive or percent change, and whether intermediate inputs make sense. For the filling pool, the model also needs to stop when capacity is reached; the teacher domain discussion omits that endpoint.

SE PDF p. 3 (printed p. 1), #1–2; pictured #1; p. 4 (printed p. 2), #3–4; TE PDF p. 9, Discuss continuation: filling-pool domain

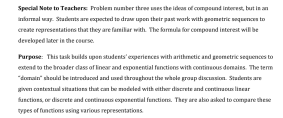

1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day n.

SE PDF p. 3 (printed p. 1), #1

Coherence: horizontal alignment

This lesson extends arithmetic/geometric sequence experience to continuous functions; lesson 2.2 then makes the domain decision explicit. Students revisit the same distinction through books and time rather than merely repeat a function label.

TE PDF p. 6, Purpose / Special Note to Teachers; SE PDF p. 3 (printed p. 1), #1–2; p. 9 (printed p. 7), #4–5


Special Note to Teachers: Problem number three uses the ideas of compound interest, but in an informal way. Students are expected to draw upon their past work with geometric sequences to create representations that they are familiar with. The formula for compound interest will be developed later in the course.

Purpose: This task builds upon students' experiences with arithmetic and geometric sequences to extend to the broader class of linear and exponential functions with continuous domains. The term "domain" should be introduced and used throughout the whole group discussion. Students are given contextual situations that can be modeled with either discrete and continuous linear functions, or discrete and continuous exponential functions. They are also asked to compare these types of functions using various representations.

TE PDF p. 6, Purpose / Special Note to Teachers

Depth of reasoning

The final comparisons require noticing what changes and what stays the same across situations and representations. They invite relational explanation beyond calculating outputs, although they do not explicitly require a general proof of equal-difference or equal-factor growth.

SE PDF p. 5 (printed p. 3), #5–7; TE PDF p. 8, Discuss: similarities and differences

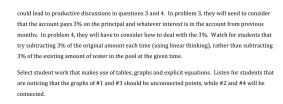
5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?

SE PDF p. 5 (printed p. 3), #5

Teacher supports

The notes identify a specific decay error: repeatedly subtracting a percent of the original amount. Teachers are directed to distinguish this linear interpretation from taking a percent of the current amount and to select contrasting representations for discussion.

TE PDF p. 8, Explore: #3–4; select student work


could lead to productive discussions in questions 3 and 4. In problem 3, they will need to consider that the account pays 3% on the principal and whatever interest is in the account from previous months. In problem 4, they will have to consider how to deal with the 3%. Watch for students that try subtracting 3% of the original amount each time (using linear thinking), rather than subtracting 3% of the existing amount of water in the pool at the given time.

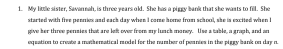
Select student work that makes use of tables, graphs and explicit equations. Listen for students that are noticing that the graphs of #1 and #3 should be unconnected points, while #2 and #4 will be connected.

TE PDF p. 8, Explore / discussion guidance

Mathematical practices

Students connect starting quantities and repeated changes to equations, tables and graphs, supporting quantitative reasoning and recognition of growth structure. Requiring all three representations is not evidence that students strategically choose tools.

SE PDF p. 3 (printed p. 1), #1; p. 4 (printed p. 2), #3–4; TE PDF p. 8, Explore / Discuss

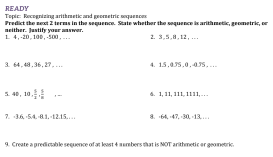

1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day n.

SE PDF p. 3 (printed p. 1), #1

Formative assessment

Ready #1–9 reveals whether students distinguish arithmetic, geometric and neither before extending sequences to functions. During exploration, monitoring the percent-decay error gives the teacher a specific issue to address in discussion.

SE PDF p. 6 (printed p. 4), READY #1–9; TE PDF p. 8, Explore: monitor #3–4


READY
Topic: Recognizing arithmetic and geometric sequences
Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.
1. 4, -20, 100, -500, . . .
2. 3, 5, 8, 12, . . .
3. 64, 48, 36, 27, . . .
4. 1.5, 0.75, 0, -0.75, . . .
5. 40, 10, $\frac{5}{2}$, $\frac{5}{8}$, . . .
6. 1, 11, 111, 1111, . . .
7. -3.6, -5.4, -8.1, -12.15, . . .
8. -64, -47, -30, -13, . . .
9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

SE PDF p. 6 (printed p. 4), READY #1–9

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Summative assessment. A separate module test was not supplied.

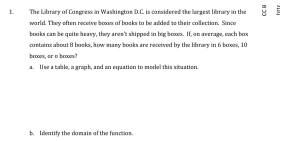
2.2 Shh! Please Be Discreet (Discrete)!

Explore lesson 2.2

Standards connection

HSF.IF.A.3 requires recognizing sequences as functions with integer-subset domains. The book and paper tasks require domain decisions and explanations, while the medicine task contrasts a continuous exponential model; this is not full coverage of every recursive-sequence demand.

SE PDF p. 8 (printed p. 6), #1a–b; p. 10 (printed p. 8), #6a–d; p. 11 (printed p. 9), #7a–d; p. 12 (printed p. 10), #8–11

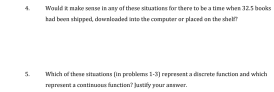

1. The Library of Congress in Washington D.C. is considered the largest library in the world. They often receive boxes of books to be added to their collection. Since books can be quite heavy, they aren't shipped in big boxes. If, on average, each box contains about 8 books, how many books are received by the library in 6 boxes, 10 boxes, or n boxes?
   a. Use a table, a graph, and an equation to model this situation.
   b. Identify the domain of the function.

SE PDF p. 8 (printed p. 6), #1a-b

Conceptual understanding

The question about fractional books forces students to relate a numerical output to what the situation permits. Comparing paper cuts with medicine decay then connects domain choices to differences in tables, graphs and equations.

SE PDF p. 9 (printed p. 7), #4–5; p. 12 (printed p. 10), #8–10

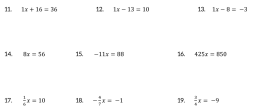

4. Would it make sense in any of these situations for there to be a time when 32.5 books had been shipped, downloaded into the computer or placed on the shelf?
5. Which of these situations (in problems 1-3) represent a discrete function and which represent a continuous function? Justify your answer.

SE PDF p. 9 (printed p. 7), #4-5

Procedural skill and practice

GO #11–19 practices one-step equations using equality-preserving operations and requires written work. The supplied examples scaffold the procedure; this supporting practice is separate from explaining discrete and continuous models.

SE PDF p. 14 (printed p. 12), GO #11–19 and worked examples

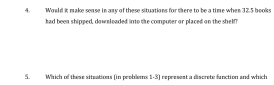

11. $1x + 16 = 36$
12. $1x - 13 = 10$
13. $1x - 8 = -3$
14. $8x = 56$
15. $-11x = 88$
16. $425x = 850$
17. $\frac{1}{5}x = 10$
18. $-\frac{1}{4}x = -1$
19. $\frac{1}{3}x = -9$

SE PDF p. 14 (printed p. 12), #11-19

Contextual application

Shelving books raises a genuine modeling choice: time can vary continuously while completed books are counted discretely. Teacher discussion makes the simplifying continuous-model assumption explicit rather than treating the time variable alone as decisive.

SE PDF p. 9 (printed p. 7), #3–5; TE PDF p. 18, Explore: competing interpretations of #3; p. 19, Discuss: shelving assumptions

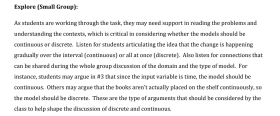

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5. Which of these situations (in problems 1-3) represent a discrete function and which represent a continuous function? Justify your answer.

SE PDF p. 9 (printed p. 7), #4-5

Coherence: horizontal alignment

The launch explicitly revisits the domain introduced in 2.1, then students use it to justify whether graph points should connect. Lesson 2.3 subsequently reuses the accumulated linear/exponential charts to support classification.

TE PDF p. 18, Launch / Explore; p. 8, Discuss: connected versus unconnected graphs; p. 28, Launch: previous-task charts


Explore (Small Group):

As students are working through the task, they may need support in reading the problems and understanding the contexts, which is critical in considering whether the models should be continuous or discrete. Listen for students articulating the idea that the change is happening gradually over the interval (continuous) or all at once (discrete). Also listen for connections that can be shared during the whole group discussion of the domain and the type of model. For instance, students may argue in #3 that since the input variable is time, the model should be continuous. Others may argue that the books aren't actually placed on the shelf continuously, so the model should be discrete. These are the type of arguments that should be considered by the class to help shape the discussion of discrete and continuous.

TE PDF p. 18, Explore (Small Group)

Depth of reasoning

Students must explain what to consider when deciding between discrete and continuous models, not simply name the model. The shelving discussion admits competing interpretations and asks students to defend assumptions about completed objects and elapsed time.

SE PDF p. 12 (printed p. 10), #9–11; TE PDF p. 18, Explore: arguments about #3; p. 19, Discuss: assumptions

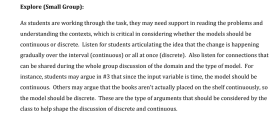

9. What needs to be considered when looking at a situation or context and deciding if it fits best with a discrete or continuous model?

SE PDF p. 12 (printed p. 10), #9

Teacher supports

The notes identify the reading/context work students may need and give two contrasting shelving arguments for discussion. This helps teachers preserve the domain decision while supporting access, though specific reading scaffolds are limited.

TE PDF p. 18, Explore: reading support and competing domain arguments


Explore (Small Group):

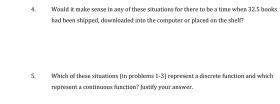
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TE PDF p. 18, Explore (Small Group)

Mathematical practices

Judging whether fractional cuts, fractional books and intermediate times make sense connects quantities to precise domain statements. The teacher's explicit assumption discussion supports modeling practice; it does not establish a required model-revision cycle.

SE PDF p. 9 (printed p. 7), #4–5; p. 10 (printed p. 8), #6c–d; p. 11 (printed p. 9), #7c–d; TE PDF p. 19, Discuss: assumptions


4. Would it make sense in any of these situations for there to be a time when 32.5 books had been shipped, downloaded into the computer or placed on the shelf?

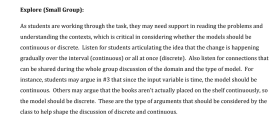
5. Which of these situations (in problems 1-3) represent a discrete function and which represent a continuous function? Justify your answer.

SE PDF p. 9 (printed p. 7), #4-5

Formative assessment

During small-group work, teachers listen for whether students use gradual versus all-at-once change to justify a domain. The contrasting time-input and completed-book arguments identify reasoning to select for whole-class discussion.

TE PDF p. 18, Explore: listen for explanations and select arguments


Explore (Small Group):

As students are working through the task, they may need support in reading the problems and understanding the contexts, which is critical in considering whether the models should be continuous or discrete. Listen for students articulating the idea that the change is happening gradually over the interval (continuous) or all at once (discrete). Also listen for connections that can be shared during the whole group discussion of the domain and the type of model. For instance, students may argue in #3 that since the input variable is time, the model should be continuous. Others may argue that the books aren't actually placed on the shelf continuously, so the model should be discrete. These are the type of arguments that should be considered by the class to help shape the discussion of discrete and continuous.

TE PDF p. 18, Explore (Small Group)

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

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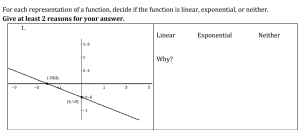
2.3 Linear, Exponential or Neither?

Explore lesson 2.3

Standards connection

The classification task directly supports HSF.LE.A.1 by distinguishing linear and exponential relationships across representations. Requiring two reasons makes growth structure visible; classification alone does not establish the contents-page claims for eventual dominance or contextual parameter interpretation.

SE PDF p. 15 (printed p. 13), Shared directions; #1–2; p. 17 (printed p. 15), #7–8; p. 20 (printed p. 18), #15–16; TE PDF p. 28, Core Standards Focus


For each representation of a function, decide if the function is linear, exponential, or neither.
Give at least 2 reasons for your answer.

1.

Linear Exponential Neither

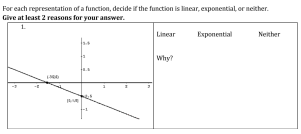
Why?

SE PDF p. 15 (printed p. 13), #1 and shared directions

Conceptual understanding

Students use graphs, equations, tables, recursive rules and verbal descriptions to identify constant differences or factors. The "neither" option requires attention to the actual relationship rather than treating every increasing function as exponential.

SE PDF p. 15 (printed p. 13), Shared directions; #1–2; p. 16 (printed p. 14), #3–5; p. 18 (printed p. 16), #9–11; p. 20 (printed p. 18), #15–16


For each representation of a function, decide if the function is linear, exponential, or neither.
Give at least 2 reasons for your answer.

1.

Linear Exponential Neither

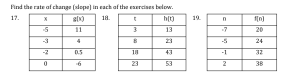
Why?

SE PDF p. 15 (printed p. 13), #1 and shared directions

Procedural skill and practice

The GO exercises calculate slope from tables, coordinate pairs and a graph, including unequal input intervals. This practices rate computation that supports linear classification; the main task additionally requires reasons.

SE PDF p. 23 (printed p. 21), GO #17–22; pictured #17–19


Find the rate of change (slope) in each of the exercises below.

17.

x	g(x)
-5	11
-3	4
-2	0.5
0	-6

18.

t	h(t)
3	13
8	23
18	43
23	53

19.

n	f(n)
-7	20
-5	24
-1	32
2	38

SE PDF p. 23 (printed p. 21), #17-19

Contextual application

The cell-phone context supplies a constant annual percent increase, which students must recognize as multiplicative growth. It supports interpreting a contextual relationship, but does not ask students to formulate or test a full model.

SE PDF p. 18 (printed p. 16), #10; p. 17 (printed p. 15), #6; p. 19 (printed p. 17), #12


10.

The number of cell phone users in Centerville as a function of years, if the number of users is increasing by 75% each year.

Linear Exponential Neither

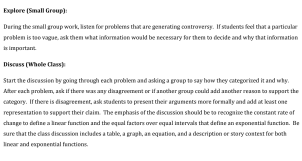
Why?

SE PDF p. 18 (printed p. 16), #10

Coherence: horizontal alignment

The launch explicitly uses charts developed in the preceding task, and students apply their growth/domain distinctions to varied representations. The following business task then uses the linear/exponential distinction to compare two income models over time.

TE PDF p. 28, Launch: charts from previous task; p. 29, Discuss: growth characteristics; SE PDF p. 24 (printed p. 22), 2.4 context and part a


Explore (Small Group):

During the small group work, listen for problems that are generating controversy. If students feel that a particular problem is too vague, ask them what information would be necessary for them to decide and why that information is important.

Discuss (Whole Class):

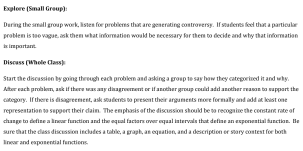
Start the discussion by going through each problem and asking a group to say how they categorized it and why. After each problem, ask if there was any disagreement or if another group could add another reason to support the category. If there is disagreement, ask students to present their arguments more formally and add at least one representation to support their claim. The emphasis of the discussion should be to recognize the constant rate of change to define a linear function and the equal factors over equal intervals that define an exponential function. Be sure that the class discussion includes a table, a graph, an equation, and a description or story content for both linear and exponential functions.

TE PDF p. 29, Explore / discussion guidance

Depth of reasoning

The shared directions require two reasons for each classification. When a representation seems ambiguous, teacher questions ask what information would decide the issue and why, turning disagreement into an argument supported by another representation.

SE PDF p. 15 (printed p. 13), Shared directions; TE PDF p. 29, Explore / Discuss


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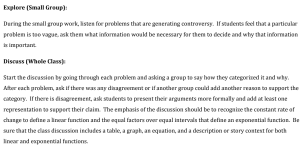
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TE PDF p. 29, Explore / discussion guidance

Teacher supports

The notes tell teachers to surface disputed cases and ask groups to strengthen their reasons with representations. Synthesis identifies equal differences and equal factors over equal intervals as the mathematical basis for classification.

TE PDF p. 29, Explore / Discuss


Explore (Small Group):

During the small group work, listen for problems that are generating controversy. If students feel that a particular problem is too vague, ask them what information would be necessary for them to decide and why that information is important.

Discuss (Whole Class):

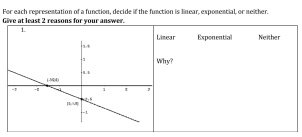
Start the discussion by going through each problem and asking a group to say how they categorized it and why. After each problem, ask if there was any disagreement or if another group could add another reason to support the category. If there is disagreement, ask students to present their arguments more formally and add at least one representation to support their claim. The emphasis of the discussion should be to recognize the constant rate of change to define a linear function and the equal factors over equal intervals that define an exponential function. Be sure that the class discussion includes a table, a graph, an equation, and a description or story context for both linear and exponential functions.

TE PDF p. 29, Explore / discussion guidance

Mathematical practices

Students justify classifications using growth structure, and teacher-led discussion asks them to evaluate disagreements. The practice demand is explanation of a relationship, not merely sorting by visual appearance.

SE PDF p. 15 (printed p. 13), Shared directions; #1; TE PDF p. 29, Discuss: formalize arguments and add representation


For each representation of a function, decide if the function is linear, exponential, or neither.
Give at least 2 reasons for your answer.

1.

Linear Exponential Neither

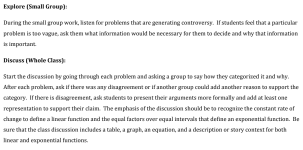
Why?

SE PDF p. 15 (printed p. 13), #1 and shared directions

Formative assessment

The teacher listens for controversial or underspecified cases, then asks what missing information would support a decision. Group explanations and disagreements reveal which growth features students are using and which need attention in synthesis.

TE PDF p. 29, Explore / Discuss


Explore (Small Group):

During the small group work, listen for problems that are generating controversy. If students feel that a particular problem is too vague, ask them what information would be necessary for them to decide and why that information is important.

Discuss (Whole Class):

Start the discussion by going through each problem and asking a group to say how they categorized it and why. After each problem, ask if there was any disagreement or if another group could add another reason to support the category. If there is disagreement, ask students to present their arguments more formally and add at least one representation to support their claim. The emphasis of the discussion should be to recognize the constant rate of change to define a linear function and the equal factors over equal intervals that define an exponential function. Be sure that the class discussion includes a table, a graph, an equation, and a description or story context for both linear and exponential functions.

TE PDF p. 29, Explore / discussion guidance

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Summative assessment. A separate module test was not supplied.

2.4 Getting Down to Business

Explore lesson 2.4

Standards connection

Students construct and compare additive and percent-growth income functions, supporting HSF.LE.A.2 and the linear-comparison portion of HSF.LE.A.3. Tables and graphs are used to examine crossover and later growth; quadratic and general polynomial comparisons are outside this task.

SE PDF p. 24 (printed p. 22), Context; parts a–b; p. 25 (printed p. 23), Parts c–e; TE PDF p. 38, Concluding discussion: end behavior


Calcu-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafest, had a net income of 2 million dollars. The management of Calcu-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafest develops a plan aimed at increasing its net income by 15% each year.

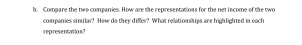
a. Create standard mathematical models (table, graph and equations) for the projected net income over time for both companies. (Attend to precision and be sure that each model is accurate and labeled properly so that it represents the situation.)

SE PDF p. 24 (printed p. 22), parts a; context

Conceptual understanding

Comparing the two representations asks students to connect a fixed increase with a percent increase. The teacher's end-behavior discussion explains why the percent increase grows with the current amount while the fixed increase stays the same.

SE PDF p. 24 (printed p. 22), Part b; TE PDF p. 38, Concluding discussion

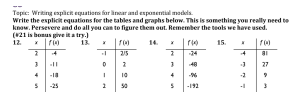

b. Compare the two companies. How are the representations for the net income of the two companies similar? How do they differ? What relationships are highlighted in each representation?

SE PDF p. 24 (printed p. 22), part b

Procedural skill and practice

The GO set requires explicit equations from tables and graphs, with constant differences or factors providing the information for rule construction. This follows the contextual modeling work, while the bonus graph presents an additional transformation challenge.

SE PDF p. 28 (printed p. 26), GO #12–21; pictured #12–15


Topic: Writing explicit equations for linear and exponential models.
Write the explicit equations for the tables and graphs below. This is something you really need to know. Persevere and do all you can to figure them out. Remember the tools we have used. (#21 is bonus give it a try.)

12.

x	$f(x)$
2	-4
3	-11
4	-18
5	-25

13.

x	$f(x)$
-1	2/5
0	2
1	10
2	50

14.

x	$f(x)$
2	-24
3	-48
4	-96
5	-192

15.

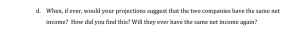
x	$f(x)$
-4	81
-3	27
-2	9
-1	3

SE PDF p. 28 (printed p. 26), #12-15

Contextual application

Students use projected income to justify an investment and determine when one company overtakes the other. Part e asks them to weigh discrete and continuous models, but the task supplies both companies' starting values and growth assumptions.

SE PDF p. 24 (printed p. 22), Context; part a; p. 25 (printed p. 23), Parts c–e; pictured part d; TE PDF p. 37, Discuss and Re-launch: model assumptions

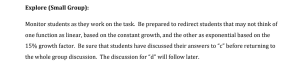

d. When, if ever, would your projections suggest that the two companies have the same net income? How did you find this? Will they ever have the same net income again?

SE PDF p. 25 (printed p. 23), part d

Coherence: horizontal alignment

The task puts the preceding classification work to use: students must recognize different growth structures before comparing their consequences. Teacher synthesis connects recursive and explicit forms, preparing for the efficient-form decisions in 2.6.

SE PDF p. 15 (printed p. 13), 2.3 shared classification directions; TE PDF p. 37, Explore / Discuss and Re-launch; SE PDF p. 38 (printed p. 36), 2.6 #1


Explore (Small Group):

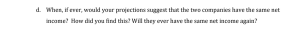
Monitor students as they work on the task. Be prepared to redirect students that may not think of one function as linear, based on the constant growth, and the other as exponential based on the 15% growth factor. Be sure that students have discussed their answers to "c" before returning to the whole group discussion. The discussion for "d" will follow later.

TE PDF p. 37, Explore (Small Group)

Depth of reasoning

The crossover question asks how students found their result and whether equality can occur again. Teacher follow-up examines later years and growth mechanisms; graphing a few values alone is not a general proof of exponential dominance.

SE PDF p. 25 (printed p. 23), Part d; TE PDF p. 37, Discuss: compare tables and graphs; p. 38, Concluding discussion

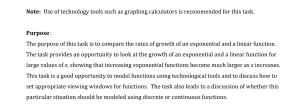

d. When, if ever, would your projections suggest that the two companies have the same net income? How did you find this? Will they ever have the same net income again?

SE PDF p. 25 (printed p. 23), part d

Teacher supports

The notes recommend graphing technology, identify viewing-window decisions and explain consistent income units. Subsequent guidance uses tables and graphs to locate crossover and discuss end behavior, with limited operational or no-technology instructions.

TE PDF p. 35, Note / Purpose; p. 36, Launch: units and scale; p. 37, Discuss and Re-launch; p. 38, Concluding discussion


Note: Use of technology tools such as graphing calculators is recommended for this task.

Purpose:
The purpose of this task is to compare the rates of growth of an exponential and a linear function. The task provides an opportunity to look at the growth of an exponential and a linear function for large values of x, showing that increasing exponential functions become much larger as x increases. This task is a good opportunity to model functions using technological tools and to discuss how to set appropriate viewing windows for functions. The task also leads to a discussion of whether this particular situation should be modeled using discrete or continuous functions.

TE PDF p. 35, Note / Purpose

Mathematical practices

Students label representations, interpret income in consistent units and discuss the assumptions behind continuous or annual models. Technology is recommended by the teacher, so its use alone does not establish strategic student tool selection.

SE PDF p. 24 (printed p. 22), Part a; p. 25 (printed p. 23), Part e; TE PDF p. 36, Launch: units and scale; p. 37, Discuss and Re-launch


Calco-rama had a net income of 5 million dollars in 2010, while a small competing company, Computafast, had a net income of 2 million dollars. The management of Calco-rama develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of Computafast develops a plan aimed at increasing its net income by 15% each year.

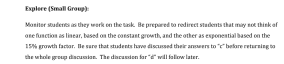
a. Create standard mathematical models (table, graph and equations) for the projected net income over time for both companies. (Attend to precision and be sure that each model is accurate and labeled properly so that it represents the situation.)

SE PDF p. 24 (printed p. 22), parts a; context

Formative assessment

During exploration, teachers check whether students distinguish a fixed increase from percent growth before comparing investment choices. The later presentation of tables and graphs exposes how students locate and interpret crossover.

TE PDF p. 37, Explore / Discuss and Re-launch


Explore (Small Group):

Monitor students as they work on the task. Be prepared to redirect students that may not think of one function as linear, based on the constant growth, and the other as exponential based on the 15% growth factor. Be sure that students have discussed their answers to "c" before returning to the whole group discussion. The discussion for "d" will follow later.

TE PDF p. 37, Explore (Small Group)

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Summative assessment. A separate module test was not supplied.

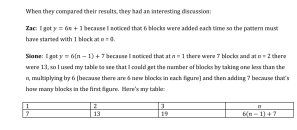
2.5 Making My Point

Explore lesson 2.5

Standards connection

The quilt dialogue supports HSA.SSE.A.1 by connecting parts of different expressions to the same growing pattern. Subsequent line tasks generalize a rule from an arbitrary point and slope; the source's A.SSE.6 label is not a valid 2010 CCSS identifier.

SE PDF p. 29 (printed p. 27), Zac and Sione dialogue/table; p. 31 (printed p. 29), #2–5; TE PDF p. 47, Core Standards Focus


When they compared their results, they had an interesting discussion:

Zac: I got $y = 6n + 1$ because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at $n = 0$.

Sione: I got $y = 6(n - 1) + 7$ because I noticed that at $n = 1$ there were 7 blocks and at $n = 2$ there were 13, so I used my table to see that I could get the number of blocks by taking one less than the n, multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

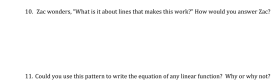
1	2	3	n
7	13	19	$6(n-1)+7$

SE PDF p. 29 (printed p. 27), quilt-pattern explanations and table

Conceptual understanding

Students compare two expressions for the quilt, then explain why equations based on different points can describe the same line. The final questions connect this flexibility to constant rate of change rather than treating point-slope form as an unexplained formula.

SE PDF p. 30 (printed p. 28), #1; p. 31 (printed p. 29), #2–5; p. 33 (printed p. 31), #10–11; TE PDF p. 49, Discuss continuation


10. Zac wonders, "What is it about lines that makes this work?" How would you answer Zac?

11. Could you use this pattern to write the equation of any linear function? Why or why not?

SE PDF p. 33 (printed p. 31), #10-11

Procedural skill and practice

Tasks #6–8 require writing line equations from slopes, points or a graph and showing a way to check each result. Homework then practices slope-intercept and point-slope forms; the pictured set shows the slope-intercept portion.

SE PDF p. 32 (printed p. 30), #6–8; p. 34 (printed p. 32), READY #1–9; pictured #1–3


Topic: Writing equations of lines

Write the equation of a line in slope-intercept form: y = mx + b, using the given information.

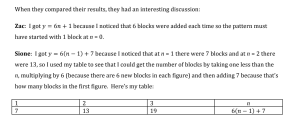
1. m = -7, b = 4 2. m = 3/8, b = -3 3. m = 16, b = -1/5

SE PDF p. 34 (printed p. 32), #1-3

Contextual application

The quilt context gives meaning to the initial block count and the number added at each stage. Students must evaluate two ways of encoding the same pattern, although the competing expressions are supplied.

SE PDF p. 29 (printed p. 27), Quilt dialogue/table; p. 30 (printed p. 28), #1


When they compared their results, they had an interesting discussion:

Zac: I got $y = 6n + 1$ because I noticed that 6 blocks were added each time so the pattern must have started with 1 block at $n = 0$.

Sione: I got $y = 6(n - 1) + 7$ because I noticed that at $n = 1$ there were 7 blocks and at $n = 2$ there were 13, so I used my table to see that I could get the number of blocks by taking one less than the n, multiplying by 6 (because there are 6 new blocks in each figure) and then adding 7 because that's how many blocks in the first figure. Here's my table:

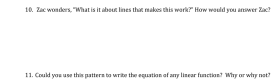
1	2	3	n
7	13	19	$6(n-1)+7$

SE PDF p. 29 (printed p. 27), quilt-pattern explanations and table

Coherence: horizontal alignment

Teacher synthesis connects the new point-based rules to previously used slope-intercept form. Lesson 2.6 then presents both forms and asks students to choose one using the available information.

TE PDF p. 49, Discuss continuation: connection to slope-intercept; SE PDF p. 33 (printed p. 31), #10–11; p. 37 (printed p. 35), 2.6 equation-form table; p. 38 (printed p. 36), 2.6 #1–2


10. Zac wonders, "What is it about lines that makes this work?" How would you answer Zac?

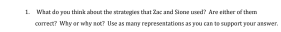
11. Could you use this pattern to write the equation of any linear function? Why or why not?

SE PDF p. 33 (printed p. 31), #10-11

Depth of reasoning

Students decide whether Zac and Sione can both be right, support that conclusion mathematically, and generalize the observed pattern. Explaining why it works for linear functions requires connecting repeated examples to constant slope.

SE PDF p. 30 (printed p. 28), #1; p. 31 (printed p. 29), #2–5; p. 33 (printed p. 31), #10–11

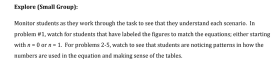

1. What do you think about the strategies that Zac and Sione used? Are either of them correct? Why or why not? Use as many representations as you can to support your answer.

SE PDF p. 30 (printed p. 28), #1

Teacher supports

Teacher notes direct attention to indexing choices and equivalent equations, then offer a derivation from the slope formula. They distinguish the immediate conceptual goal from a derivation that may come later.

TE PDF p. 48, Explore / Discuss; p. 49, Discuss continuation


Explore (Small Group):

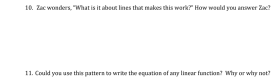
Monitor students as they work through the task to see that they understand each scenario. In problem #1, watch for students that have labeled the figures to match the equations; either starting with $n = 0$ or $n = 1$. For problems 2-5, watch to see that students are noticing patterns in how the numbers are used in the equation and making sense of the tables.

TE PDF p. 48, Explore (Small Group)

Mathematical practices

Students critique two arguments, recognize equivalent structure and generalize a point-based rule. The explicit "why any linear function" questions provide stronger evidence of generalization than routine substitution alone.

SE PDF p. 30 (printed p. 28), #1; p. 31 (printed p. 29), #2–5; p. 33 (printed p. 31), #10–11


10. Zac wonders, "What is it about lines that makes this work?" How would you answer Zac?

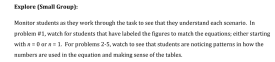
11. Could you use this pattern to write the equation of any linear function? Why or why not?

SE PDF p. 33 (printed p. 31), #10-11

Formative assessment

Monitoring checks whether students align figure numbering with the given expressions and make sense of the tables. Comparing apparently different equations then gives the teacher evidence of whether students understand equivalence and point-based structure.

TE PDF p. 48, Explore / Discuss


Explore (Small Group):

Monitor students as they work through the task to see that they understand each scenario. In problem #1, watch for students that have labeled the figures to match the equations; either starting with $n = 0$ or $n = 1$. For problems 2-5, watch to see that students are noticing patterns in how the numbers are used in the equation and making sense of the tables.

TE PDF p. 48, Explore (Small Group)

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Summative assessment. A separate module test was not supplied.

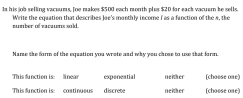
2.6 Form Follows Function

Explore lesson 2.6

Standards connection

HSF.LE.A.2 requires constructing linear and exponential functions from varied information. Tasks #1–5 require rules from contexts, a graph or a sequence, while #6–9 use symbolic forms to graph; the source limits its graphing focus to linear and exponential functions.

SE PDF p. 38 (printed p. 36), #1–2; p. 39 (printed p. 37), Both source items labeled #3; #4; p. 40 (printed p. 38), #5–6; p. 41 (printed p. 39), #7–9; TE PDF p. 58, Core Standards Focus



1. In his job selling vacuums, Joe makes $500 each month plus $20 for each vacuum he sells. Write the equation that describes Joe's monthly income I as a function of the x, the number of vacuums sold.

Name the form of the equation you wrote and why you chose to use that form.

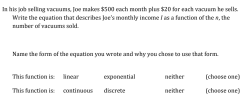
This function is: linear exponential neither (choose one)
This function is: continuous discrete neither (choose one)

SE PDF p. 38 (printed p. 36), #1

Conceptual understanding

Students explain why a chosen equation form fits the information given and identify features that help graph a function. The teacher comparison of methods connects efficiency to mathematical structure, rather than merely attaching names to forms.

SE PDF p. 38 (printed p. 36), Introduction; #1–2; p. 40 (printed p. 38), #5–6; p. 41 (printed p. 39), #7–9; TE PDF p. 59, Explore / Discuss



1. In his job selling vacuums, Joe makes $500 each month plus $20 for each vacuum he sells. Write the equation that describes Joe's monthly income I as a function of the x, the number of vacuums sold.

Name the form of the equation you wrote and why you chose to use that form.

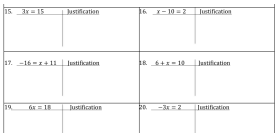
This function is: linear exponential neither (choose one)
This function is: continuous discrete neither (choose one)

SE PDF p. 38 (printed p. 36), #1

Procedural skill and practice

Students construct equations and graphs, then compare which forms use the given information efficiently. The GO set separately maintains equation-solving with equality justifications; feedback guidance centers on class comparison of methods.

SE PDF p. 38 (printed p. 36), #1–2; p. 43 (printed p. 41), SET #9–10; p. 44 (printed p. 42), SET #11–14; p. 45 (printed p. 43), GO #15–20; TE PDF p. 59, Discuss: compare efficiency



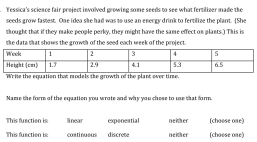
15. $3x = 15$ Justification
16. $x - 10 = 2$ Justification
17. $-16 = x + 11$ Justification
18. $-5 + x = 10$ Justification
19. $6x = 18$ Justification
20. $-3x = 2$ Justification

SE PDF p. 45 (printed p. 43), #15-20

Contextual application

The plant-data task asks students to infer a growth rule from height measurements and explain their equation-form choice. Other situations connect rules to income, population and savings; most quantities and relationships are still tightly specified.

SE PDF p. 40 (printed p. 38), #5; p. 38 (printed p. 36), #1; p. 39 (printed p. 37), #4; p. 43 (printed p. 41), #9



5. Yessica's science fair project involved growing some seeds to see what fertilizer made the seeds grow fastest. One idea she had was to use an energy drink to fertilize the plant. (She thought that if they make people perky, they might have the same effect on plants.) This is the data that shows the growth of the seed each week of the project.

Week	1	2	3	4	5
Height (cm)	1.7	2.9	4.1	5.3	6.5

Write the equation that models the growth of the plant over time.

Name the form of the equation you wrote and why you chose to use that form.

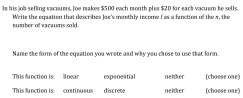
This function is: linear exponential neither (choose one)
This function is: continuous discrete neither (choose one)

SE PDF p. 40 (printed p. 38), #5

Coherence: horizontal alignment

The opening table gathers forms developed earlier, including the point-based form explained in 2.5. Students now use those forms to select efficient methods and graph from visible equation features.

SE PDF p. 33 (printed p. 31), 2.5 #10–11; p. 37 (printed p. 35), Equation-form table; p. 38 (printed p. 36), Introduction; #1–2; TE PDF p. 59, Discuss: graph from information in each form



1. In his job selling vacuums, Joe makes $500 each month plus $20 for each vacuum he sells. Write the equation that describes Joe's monthly income I as a function of the x, the number of vacuums sold.

Name the form of the equation you wrote and why you chose to use that form.

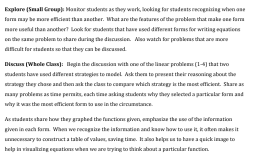
This function is: linear exponential neither (choose one)
This function is: continuous discrete neither (choose one)

SE PDF p. 38 (printed p. 36), #1

Depth of reasoning

Repeated requests to explain a form choice make students connect problem information to a method. Teacher discussion compares different valid approaches for efficiency, rather than accepting a correct equation as sufficient explanation.

SE PDF p. 38 (printed p. 36), #1–2; p. 39 (printed p. 37), Both #3 items; #4; p. 40 (printed p. 38), #5; TE PDF p. 59, Explore / Discuss



Explore (Small Group): Monitor students as they work, looking for students recognizing when one form may be more efficient than another. What are the features of the problem that make one form more useful than another? Look for students that have used different forms for writing equations on the same problem to share during the discussion. Also watch for problems that are more difficult for students so that they can be discussed.

Discuss (Whole Class): Begin the discussion with one of the linear problems (1–4) that two students have used different strategies to model. Ask them to present their reasoning about the strategy they chose and then ask the class to compare which strategy is the most efficient. Share as many problems as time permits, each time asking students why they selected a particular form and why it was the most efficient form to use in the circumstance.

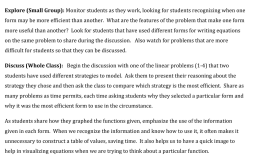
As students share how they graphed the functions given, emphasize the use of the information given in each form. When we recognize the information and know how to use it, it often makes it unnecessary to construct a table of values, saving time. It also helps us to have a quick image to help in visualizing equations when we are trying to think about a particular function.

TE PDF p. 59, Explore / discussion guidance

Teacher supports

The notes identify what to monitor: whether students recognize when one form is more useful and which problems remain difficult. Discussion then compares student strategies and explains when structural graphing can replace a table of values.

TE PDF p. 59, Explore / Discuss



Explore (Small Group): Monitor students as they work, looking for students recognizing when one form may be more efficient than another. What are the features of the problem that make one form more useful than another? Look for students that have used different forms for writing equations on the same problem to share during the discussion. Also watch for problems that are more difficult for students so that they can be discussed.

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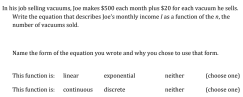
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TE PDF p. 59, Explore / discussion guidance

Mathematical practices

Students justify method choices, connect equation features to graphs and justify equality operations in homework. Choosing an equation form is credited here as structural/method reasoning, not as evidence of strategic tool choice.

SE PDF p. 38 (printed p. 36), #1–2; p. 41 (printed p. 39), #7–9; p. 45 (printed p. 43), #15–20; TE PDF p. 59, Discuss: compare methods



1. In his job selling vacuums, Joe makes $500 each month plus $20 for each vacuum he sells. Write the equation that describes Joe's monthly income I as a function of the x, the number of vacuums sold.

Name the form of the equation you wrote and why you chose to use that form.

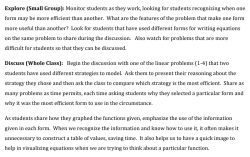
This function is: linear exponential neither (choose one)
This function is: continuous discrete neither (choose one)

SE PDF p. 38 (printed p. 36), #1

Formative assessment

The teacher monitors form selection and identifies difficult problems while students work. Selecting two different approaches to the same problem allows discussion to reveal whether choices are justified and efficient.

TE PDF p. 59, Explore / Discuss



Explore (Small Group): Monitor students as they work, looking for students recognizing when one form may be more efficient than another. What are the features of the problem that make one form more useful than another? Look for students that have used different forms for writing equations on the same problem to share during the discussion. Also watch for problems that are more difficult for students so that they can be discussed.

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As students share how they graphed the functions given, emphasize the use of the information given in each form. When we recognize the information and know how to use it, it often makes it unnecessary to construct a table of values, saving time. It also helps us to have a quick image to help in visualizing equations when we are trying to think about a particular function.

TE PDF p. 59, Explore / discussion guidance

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Summative assessment. A separate module test was not supplied.

Source credit

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