Algebra 2 - Sequences, Polynomials & Rational Functions

Illustrative Mathematics, original Kendall Hunt first-edition pathway (2019); public teacher sources retrieved October 25, 2026 and may include publisher corrections

Review summary

62%

Review completeness

36 of 58 criteria provisionally judged



Gateway 1

Standards and progression

30%

3/10 criteria judged

Gateway 2

Rigor and mathematical practice

92%

11/12 criteria judged

Gateway 3

Teaching and participation

81%

17/21 criteria judged

Gateway 4

Assessment and implementation

33%

5/15 criteria judged

These percentages describe review completeness, not curriculum quality. Quality rating pending.

Key findings

Sequence representations develop into polynomial division, rational expressions and finite sums.

Teacher guidance supports mathematical discussion, misconception responses, language development and purposeful group work.

Protected assessments and incomplete standards mapping limit the review.

Reviewed scope

Units 1-2, 37 lessons; 74 student task/practice PDFs, public teacher lessons, preparation and supporting guides. Provisional findings; unassessed criteria are not zeros. No whole-course rating.

Explore the interactive review

1.1 A Towering Sequence

Explore lesson 1.1

Standards connection

Students solve Tower of Hanoi cases, explain how a smaller solution informs a larger case, and predict larger minimum counts. The rules constrain allowable moves; a trial alone does not prove minimality. HSF.BF.A.1a: recursive-process introduction from explicit puzzle constraints, not full arithmetic/geometric sequence coverage.

SE 1.1 task · PDF p. 2, Activity1.2 #1–5



SE 1.1 task · PDF p. 2, Activity 1.2

Conceptual understanding

Students connect the minimum moves for a smaller Hanoi tower with the work needed for a larger tower. Reusing a smaller solution develops a recursive relationship; finding a successful trial alone does not prove it is minimal.

SE 1.1 task · PDF p. 2, Activity1.2 #1–5

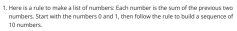


SE 1.1 task · PDF p. 2, Activity 1.2

Procedural skill and practice

Practice #1 generates ten terms by adding the preceding two terms. This exercises carrying out a recursive rule with two initial values, not arithmetic or geometric sequence fluency.

SE 1.1 practice · PDF p. 1, Practice #1



SE 1.1 practice · PDF p. 1, #1

Coherence: horizontal alignment

The warm-up generates a list from a stated rule; the Hanoi investigation then asks students to discover a rule from constrained moves. The change from executing a rule to explaining its origin is the local connection.

SE 1.1 task · PDF p. 2, Activity1.2 #1–5; p. 1, Warm-up 1.1



SE 1.1 task · PDF p. 2, Activity 1.2

Depth of reasoning

Activity 1.2 asks how the three-disc solution can help solve four discs and why a five-disc prediction is plausible. The explanation must respect both movement rules; a pattern in recorded trials is not yet an optimality proof.

SE 1.1 task · PDF p. 2, Activity1.2 #1–5; TE Student task, anticipated reasoning and Activity Synthesis

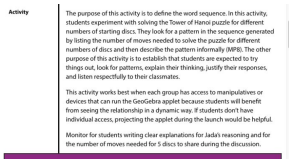


SE 1.1 task · PDF p. 2, Activity 1.2

Teacher supports

The Hanoi guidance offers manipulatives or a dynamic applet and suggests projection when individual devices are unavailable. These alternatives retain the move constraints and pattern investigation rather than replace them with a supplied formula.

TE 1.2 Activity; SE 1.1 task · PDF p. 2, Activity1.2 #1–5



TE 1.2 Activity

Mathematical practices

MP8 is supported when students reuse a solved smaller tower to predict a larger case and check the conjecture against moves. Applet use alone is not evidence of strategic tool selection.

SE 1.1 task · PDF p. 2, Activity1.2 #1–5; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 1.1 task · PDF p. 2, Activity 1.2

Formative assessment

The teacher monitors explanations of how the smaller puzzle informs the larger one, then compares the smallest move counts groups found. A larger count signals a need to revisit the strategy; the public guidance does not establish the quality of the protected exit item.

SE 1.1 task · PDF p. 2, Activity1.2 #1–5; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 1.1 task · PDF p. 2, Activity 1.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

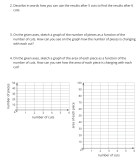
1.2 Introducing Geometric Sequences

Explore lesson 1.2

Standards connection

Students extend the paper-cutting process verbally and sketch and interpret separate count and single-piece-area graphs. Growth and decay are different quantities, while total area stays invariant. HSF.BF.A.2: multiplicative sequence structure in context; full explicit/recursive translation not required by this task.

SE 1.2 task · PDF p. 2, Activity2.2 #2–4 (table and setup on PDFp1)

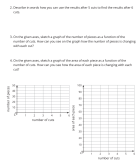


SE 1.2 task · PDF p. 2, Activity 2.2

Conceptual understanding

Paper slicing produces two related geometric sequences: piece counts grow while each piece shrinks. Students must connect each quantity with its own table and graph, rather than describe both as the same growth.

SE 1.2 task · PDF p. 2, Activity2.2 #2–4 (table and setup on PDFp1)



SE 1.2 task · PDF p. 2, Activity 2.2

Procedural skill and practice

Practice #1 extends the geometric sequence beginning with two and four. The calculation practices repeated multiplication; the later practice items broaden growth factors and decay cases.

SE 1.2 practice · PDF p. 1, Practice #1

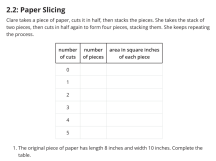


SE 1.2 practice · PDF p. 1, #1

Contextual application

Folding changes the number and area of paper pieces in different ways. Students must identify which quantity a sequence describes and explain the effect of one additional fold.

SE 1.2 task · PDF p. 1, Activity 2.2

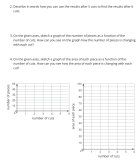


SE 1.2 task · PDF p. 1, Activity 2.2

Coherence: horizontal alignment

The folded-paper investigation supplies the counts later revisited in Lesson 8 as explicit rules. Here students explain each fold; the later task asks how a chosen index changes the formula.

SE 1.2 task · PDF p. 2, Activity2.2 #2–4 (table and setup on PDFp1); 1.8 task · PDF p. 1, Activity 8.2

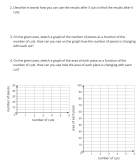


SE 1.2 task · PDF p. 2, Activity 2.2

Depth of reasoning

Students explain how the table after five cuts predicts the sixth and how each graph shows change. Their account must distinguish a doubling count from a halving area, not merely state that a pattern exists.

SE 1.2 task · PDF p. 2, Activity2.2 #2–4 (table and setup on PDFp1); TE Student task, anticipated reasoning and Activity Synthesis

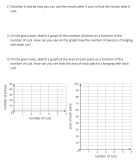


SE 1.2 task · PDF p. 2, Activity 2.2

Teacher supports

The launch permits actual cutting or a demonstration, and the teacher distinguishes what area means from the multiplication used to calculate it. That support targets the quantity being modeled while leaving students to interpret the sequences.

TE 2.2 Launch and Anticipated Misconceptions; SE 1.2 task · PDF p. 2, Activity2.2 #2–4 (table and setup on PDFp1)

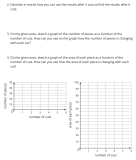


SE 1.2 task · PDF p. 2, Activity 2.2

Mathematical practices

MP2 is supported by translating cuts, counts and square inches into numerical and graphical representations. The process is supplied, so this is not evidence of independently choosing a model.

SE 1.2 task · PDF p. 2, Activity2.2 #2–4 (table and setup on PDFp1); TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

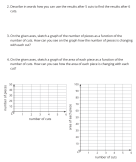


SE 1.2 task · PDF p. 2, Activity 2.2

Formative assessment

The teacher selects student graphs and asks where each growth factor is visible. Comparing the two graphs can expose confusion between number of pieces and area of a piece before formal sequence equations are expected.

SE 1.2 task · PDF p. 2, Activity2.2 #2–4 (table and setup on PDFp1); TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 1.2 task · PDF p. 2, Activity 2.2

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Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

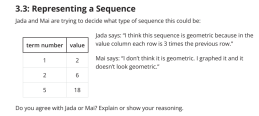
1.3 Different Types of Sequences

Explore lesson 1.3

Standards connection

Students judge whether tripling between displayed table rows establishes a geometric sequence. The last displayed term index skips three positions, so adjacent displayed values are not consecutive terms. HSF.BF.A.2/HSF.LE.A.2 contribution: distinguish constant per-step factors from ratios over unequal input gaps.

SE 1.3 task · PDF p. 2, Activity3.3, Jada/Mai claim and table

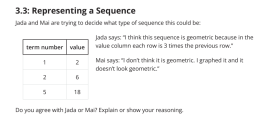


SE 1.3 task · PDF p. 2, Activity 3.3

Conceptual understanding

The lesson contrasts additive, multiplicative and other sequence patterns. In Activity 3.3, the skipped term numbers mean that ratios between neighboring table rows do not automatically describe consecutive sequence terms.

SE 1.3 task · PDF p. 2, Activity3.3, Jada/Mai claim and table

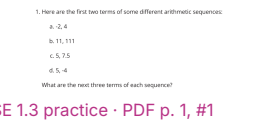


SE 1.3 task · PDF p. 2, Activity 3.3

Procedural skill and practice

Practice #1 extends four arithmetic sequences using their common differences. Negative and fractional values broaden execution beyond a single positive-integer example.

SE 1.3 practice · PDF p. 1, Practice #1

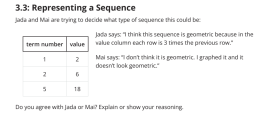


SE 1.3 practice · PDF p. 1, #1

Coherence: horizontal alignment

The preceding geometric-sequence work uses a repeated multiplier; this lesson contrasts that structure with repeated addition. Students must distinguish the relationship between successive terms from changes in the input labels.

SE 1.3 task · PDF p. 2, Activity3.3, Jada/Mai claim and table; 1.2 task · PDF p. 3, Activity 2.3

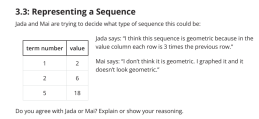


SE 1.3 task · PDF p. 2, Activity 3.3

Depth of reasoning

Students evaluate Jada's and Mai's claims using the term-number column as well as the outputs. The missing input positions are decisive; tripling between displayed rows is insufficient justification.

SE 1.3 task · PDF p. 2, Activity3.3, Jada/Mai claim and table; TE Student task, anticipated reasoning and Activity Synthesis

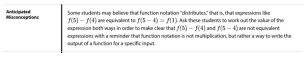


SE 1.3 task · PDF p. 2, Activity 3.3

Teacher supports

The function-notation warm-up anticipates the mistake of treating function notation like multiplication or distributing it across subtraction. Teachers ask students to evaluate both proposed expressions to reveal their different meanings.

TE 3.1 Anticipated Misconceptions; SE 1.3 task · PDF p. 2, Activity3.3, Jada/Mai claim and table


Anticipated Misconceptions

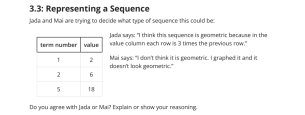
Some students may believe that function notation "distributes," that is, that expressions like $f(5)-f(4)$ are equivalent to $f(5-4)=f(1)$. Ask these students to work out the value of the expression both ways in order to make clear that $f(5)-f(4)$ and $f(5-4)$ are not equivalent expressions with a reminder that function notation is not multiplication, but rather a way to write the output of a function for a specific input.

TE 3.1 Anticipated Misconceptions

Mathematical practices

MP3 is supported by judging competing claims about a sequence and explaining which information makes a claim valid. A graph's appearance or an output ratio needs to be checked against the input spacing.

SE 1.3 task · PDF p. 2, Activity3.3, Jada/Mai claim and table; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


3.3: Representing a Sequence

Jada and Mai are trying to decide what type of sequence this could be:

term number	value
1	2
2	6
5	18

Jada says: "I think this sequence is geometric because in the value column each row is 3 times the previous row."

Mai says: "I don't think it is geometric. I graphed it and it doesn't look geometric."

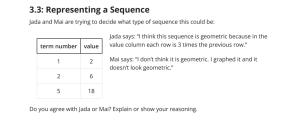
Do you agree with Jada or Mai? Explain or show your reasoning.

SE 1.3 task · PDF p. 2, Activity 3.3

Formative assessment

The teacher uses the warm-up to hear existing function-notation strategies before students formalize recurrences. A mismatched value guides a targeted notation comparison rather than only marking an answer wrong.

SE 1.3 task · PDF p. 2, Activity3.3, Jada/Mai claim and table; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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Mai says: "I don't think it is geometric. I graphed it and it doesn't look geometric."

Do you agree with Jada or Mai? Explain or show your reasoning.

SE 1.3 task · PDF p. 2, Activity 3.3

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Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

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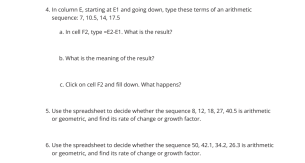
1.4 Using Technology to Work with Sequences

Explore lesson 1.4

Standards connection

Students interpret repeated differences and classify sequences with a spreadsheet. The tool and method are prescribed, so the task shows useful technology use rather than independent tool selection. HSF.BF.A.2 contribution: interpret recursive additive/multiplicative processes; technology routine does not automatically establish MP5 choice.

SE 1.4 task · PDF p. 3, Activity4.2 #4–6; earlier spreadsheet setup PDFp2


4. In column E, starting at E1 and going down, type these terms of an arithmetic sequence: 7, 10.5, 14, 17.5

a. In cell F2, type =E2-E1. What is the result?

b. What is the meaning of the result?

c. Click on cell F2 and fill down. What happens?

5. Use the spreadsheet to decide whether the sequence 8, 12, 18, 27, 40.5 is arithmetic or geometric, and find its rate of change or growth factor.

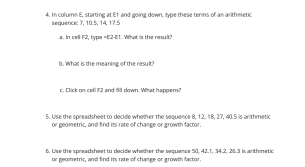
6. Use the spreadsheet to decide whether the sequence 50, 42.1, 34.2, 26.3 is arithmetic or geometric, and find its rate of change or growth factor.

SE 1.4 task · PDF p. 3, Activity 4.2

Conceptual understanding

Spreadsheet formulas connect repeated addition or multiplication with sequence generation. Students change starting values and rules, then interpret what the differences or ratios reveal about the sequence.

SE 1.4 task · PDF p. 3, Activity4.2 #4–6; earlier spreadsheet setup PDFp2


4. In column E, starting at E1 and going down, type these terms of an arithmetic sequence: 7, 10.5, 14, 17.5

a. In cell F2, type =E2-E1. What is the result?

b. What is the meaning of the result?

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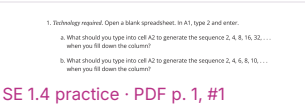
6. Use the spreadsheet to decide whether the sequence 50, 42.1, 34.2, 26.3 is arithmetic or geometric, and find its rate of change or growth factor.

SE 1.4 task · PDF p. 3, Activity 4.2

Procedural skill and practice

Practice #1 asks students to write spreadsheet formulas for doubling and constant-increment sequences. The task requires a cell reference that continues correctly when filled down, not just typing the desired terms.

SE 1.4 practice · PDF p. 1, Practice #1


1. *Technology required.* Open a blank spreadsheet. In A1, type 2 and enter.

a. What should you type into cell A2 to generate the sequence 2, 4, 8, 16, 32, . . . when you fill down the column?

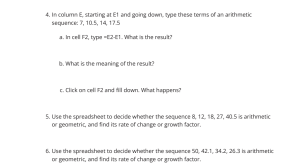
b. What should you type into cell A2 to generate the sequence 2, 4, 6, 8, 10, . . . when you fill down the column?

SE 1.4 practice · PDF p. 1, #1

Coherence: horizontal alignment

Lesson 3 distinguishes arithmetic and geometric lists; this lesson tests those structures in spreadsheet columns. Computing differences and ratios gives a systematic check on a classification previously made by inspection.

SE 1.4 task · PDF p. 3, Activity4.2 #4–6; earlier spreadsheet setup PDFp2; 1.3 task · PDF p. 1, Activity 3.2


4. In column E, starting at E1 and going down, type these terms of an arithmetic sequence: 7, 10.5, 14, 17.5

a. In cell F2, type =E2-E1. What is the result?

b. What is the meaning of the result?

c. Click on cell F2 and fill down. What happens?

5. Use the spreadsheet to decide whether the sequence 8, 12, 18, 27, 40.5 is arithmetic or geometric, and find its rate of change or growth factor.

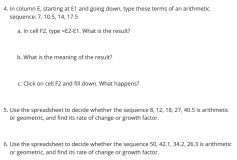
6. Use the spreadsheet to decide whether the sequence 50, 42.1, 34.2, 26.3 is arithmetic or geometric, and find its rate of change or growth factor.

SE 1.4 task · PDF p. 3, Activity 4.2

Depth of reasoning

Students must explain the meaning of spreadsheet ratios and differences and use them to classify new lists. A constant output column is useful only when students connect it with the common difference or growth factor.

SE 1.4 task · PDF p. 3, Activity4.2 #4–6; earlier spreadsheet setup PDFp2; TE Student task, anticipated reasoning and Activity Synthesis


4. In column E, starting at E1 and going down, type these terms of an arithmetic sequence: 7, 10.5, 14, 17.5

a. In cell F2, type =E2-E1. What is the result?

b. What is the meaning of the result?

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5. Use the spreadsheet to decide whether the sequence 8, 12, 18, 27, 40.5 is arithmetic or geometric, and find its rate of change or growth factor.

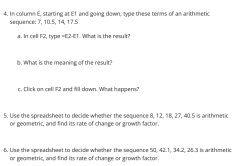
6. Use the spreadsheet to decide whether the sequence 50, 42.1, 34.2, 26.3 is arithmetic or geometric, and find its rate of change or growth factor.

SE 1.4 task · PDF p. 3, Activity 4.2

Teacher supports

The teacher explicitly teaches cell addresses, formula entry, fill-down and plotting a table. A physical demonstration is suggested to connect the new interface with familiar function representations.

TE 4.1 Launch; 4.3 Launch; SE 1.4 task · PDF p. 3, Activity4.2 #4–6; earlier spreadsheet setup PDFp2


4. In column E, starting at E1 and going down, type these terms of an arithmetic sequence: 7, 10.5, 14, 17.5

a. In cell F2, type =E2-E1. What is the result?

b. What is the meaning of the result?

c. Click on cell F2 and fill down. What happens?

5. Use the spreadsheet to decide whether the sequence 8, 12, 18, 27, 40.5 is arithmetic or geometric, and find its rate of change or growth factor.

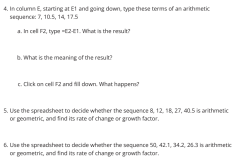
6. Use the spreadsheet to decide whether the sequence 50, 42.1, 34.2, 26.3 is arithmetic or geometric, and find its rate of change or growth factor.

SE 1.4 task · PDF p. 3, Activity 4.2

Mathematical practices

MP8 is supported when repeated calculations expose a stable additive or multiplicative rule. The spreadsheet and graphing tools are prescribed here, so their use does not establish MP5 choice.

SE 1.4 task · PDF p. 3, Activity4.2 #4–6; earlier spreadsheet setup PDFp2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


4. In column E, starting at E1 and going down, type these terms of an arithmetic sequence: 7, 10.5, 14, 17.5

a. In cell F2, type =E2-E1. What is the result?

b. What is the meaning of the result?

c. Click on cell F2 and fill down. What happens?

5. Use the spreadsheet to decide whether the sequence 8, 12, 18, 27, 40.5 is arithmetic or geometric, and find its rate of change or growth factor.

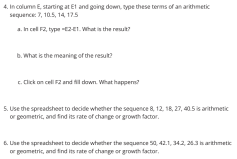
6. Use the spreadsheet to decide whether the sequence 50, 42.1, 34.2, 26.3 is arithmetic or geometric, and find its rate of change or growth factor.

SE 1.4 task · PDF p. 3, Activity 4.2

Formative assessment

The synthesis compares students' edited tables and graphs, including a relation that is not a function. This lets the teacher check whether students understand what their changes do, rather than infer understanding from successful clicks.

SE 1.4 task · PDF p. 3, Activity4.2 #4–6; earlier spreadsheet setup PDFp2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


4. In column E, starting at E1 and going down, type these terms of an arithmetic sequence: 7, 10.5, 14, 17.5

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SE 1.4 task · PDF p. 3, Activity 4.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

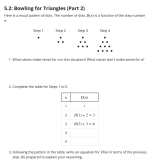
1.5 Sequences are Functions

Explore lesson 1.5

Standards connection

Students interpret valid step numbers, complete triangular-dot counts and write a recurrence in terms of the previous step. The pictured dots model a sequence that is neither arithmetic nor geometric. HSF.IF.A.3 and HSF.BF.A.1a: discrete function domain and recursive process from visual growth.

SE 1.5 task · PDF p. 2, Activity5.2 #1–3


5.2: Bowling for Triangles (Part 2)

Here is a visual pattern of dots. The number of dots, $D(n)$ is a function of the step number, n.

Step 1 Step 2 Step 3 Step 4

1. What values make sense for n? Why?

2. Complete the table for Steps 1 to 5.

n	$D(n)$
1	1
2	$D(1)+2=3$
3	$D(2)+3=6$
4	
5	

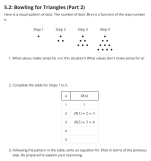
3. Following the pattern in the table, write an equation for $D(n)$ in terms of the previous step. Be prepared to explain your reasoning.

SE 1.5 task · PDF p. 2, Activity 5.2

Conceptual understanding

The triangular-dot pattern makes sequence position an input and dot count an output. Students identify sensible integer inputs and express the next stage using the preceding stage; this pattern is not itself arithmetic or geometric.

SE 1.5 task · PDF p. 2, Activity5.2 #1–3


5.2: Bowling for Triangles (Part 2)

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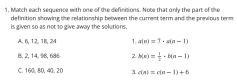
3. Following the pattern in the table, write an equation for $D(n)$ in terms of the previous step. Be prepared to explain your reasoning.

SE 1.5 task · PDF p. 2, Activity 5.2

Procedural skill and practice

Practice #1 matches sequences with the recurrence part of definitions. The source intentionally omits initial conditions for the matching exercise; it is not presenting those choices as complete definitions.

SE 1.5 practice · PDF p. 1, Practice #1


1. Match each sequence with one of the definitions. Note that only the part of the definition showing the relationship between the current term and the previous term is given so as not to give away the solutions.

A. 6, 12, 18, 24 — 1. $a(n) = 7 \cdot a(n-1)$

B. 2, 14, 98, 686 — 2. $b(n) = \frac{1}{2} \cdot b(n-1)$

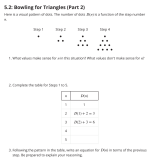
C. 160, 80, 40, 20 — 3. $c(n) = c(n-1) + 6$

SE 1.5 practice · PDF p. 1, #1

Coherence: horizontal alignment

The familiar list beginning with thirty and forty returns as a recurrence. Students now name both the initial value and the rule for obtaining the next term, making the earlier classification operational.

SE 1.5 task · PDF p. 2, Activity5.2 #1–3; 1.3 task · PDF p. 1, Activity 3.2

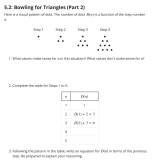


SE 1.5 task · PDF p. 2, Activity 5.2

Depth of reasoning

Students explain what the current-step and previous-step notation mean in the pattern. A valid recurrence must fit the pictured counts and identify where the sequence starts.

SE 1.5 task · PDF p. 2, Activity5.2 #1–3; TE Student task, anticipated reasoning and Activity Synthesis

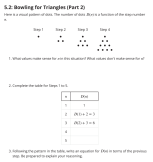


SE 1.5 task · PDF p. 2, Activity 5.2

Teacher supports

The teacher connects words about the pattern with function notation and suggests a written verbal rule as an intermediate step. A persistent class display then records arithmetic and geometric examples for later lessons.

TE 5.3 Anticipated Misconceptions; Lesson Synthesis; SE 1.5 task · PDF p. 2, Activity5.2 #1–3

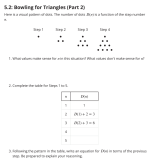


SE 1.5 task · PDF p. 2, Activity 5.2

Mathematical practices

MP2 is supported by interpreting a symbolic input as a stage number instead of any real number. Students connect the abstract recurrence with the pictured quantities and permitted domain.

SE 1.5 task · PDF p. 2, Activity5.2 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

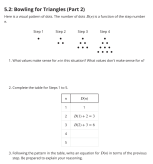


SE 1.5 task · PDF p. 2, Activity 5.2

Formative assessment

Teachers compare students' recursive definitions and attend to both the first value and subsequent rule. This can distinguish a notation difficulty from misunderstanding how the pattern changes.

SE 1.5 task · PDF p. 2, Activity5.2 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 1.5 task · PDF p. 2, Activity 5.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

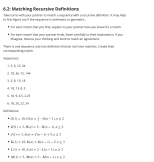
1.6 Representing Sequences

Explore lesson 1.6

Standards connection

Partners explain recurrence/sequence matches, listen to and evaluate explanations, and create missing matches. Matching requires initial value as well as additive/multiplicative rule. HSF.BF.A.2: connect recursive notation and sequence terms; the task does not yet independently require every explicit formula.

SE 1.6 task · PDF p. 2, Activity6.2 matching directions, sequences and definitions

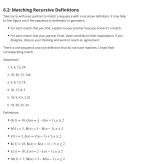


SE 1.6 task · PDF p. 2, Activity 6.2

Conceptual understanding

Matching lists and recurrences requires both a starting value and a rule. Two sequences can share doubling while still need different initial conditions, and the unmatched entries require students to create a representation.

SE 1.6 task · PDF p. 2, Activity6.2 matching directions, sequences and definitions

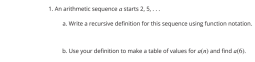


SE 1.6 task · PDF p. 2, Activity 6.2

Procedural skill and practice

Practice #1 asks for an arithmetic recurrence and a table generated from it. Students must use the stated initial terms consistently when evaluating later positions.

SE 1.6 practice · PDF p. 1, Practice #1


1. An arithmetic sequence a starts 2, 5, . . .

a. Write a recursive definition for this sequence using function notation.

b. Use your definition to make a table of values for $a(n)$ and find $a(6)$.

SE 1.6 practice · PDF p. 1, #1

Coherence: horizontal alignment

The preceding recurrence work separates the starting value from the repeated operation. Matching rules and lists here tests whether students preserve both pieces rather than matching only a common difference or ratio.

SE 1.6 task · PDF p. 2, Activity6.2 matching directions, sequences and definitions; 1.5 task · PDF p. 3, Activity 5.3


6.2: Matching Recursive Definitions

SE 1.6 task · PDF p. 2, Activity 6.2

Depth of reasoning

Partners explain a match, evaluate the explanation and create the missing counterpart. Checking only whether a rule adds or multiplies is insufficient when initial values differ.

SE 1.6 task · PDF p. 2, Activity6.2 matching directions, sequences and definitions; TE Student task, anticipated reasoning and Activity Synthesis


6.2: Matching Recursive Definitions

SE 1.6 task · PDF p. 2, Activity 6.2

Teacher supports

The preparation identifies this lesson as optional when students are already proficient with notation, tables and graphs. For learners needing it, the guidance offers chunked matching and graph paper while preserving the representation goal.

TE Lesson Narrative; SE 1.6 task · PDF p. 2, Activity6.2 matching directions, sequences and definitions


6.2: Matching Recursive Definitions

SE 1.6 task · PDF p. 2, Activity 6.2

Mathematical practices

MP3 is supported by the explain–listen–disagree protocol for each match. Students must resolve mathematical differences rather than merely divide the cards between partners.

SE 1.6 task · PDF p. 2, Activity6.2 matching directions, sequences and definitions; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


6.2: Matching Recursive Definitions

SE 1.6 task · PDF p. 2, Activity 6.2

Formative assessment

The teacher asks why two doubling sequences need different recursive definitions and listens to the created missing match. Their explanations reveal whether initial conditions are being treated as part of the definition.

SE 1.6 task · PDF p. 2, Activity6.2 matching directions, sequences and definitions; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


6.2: Matching Recursive Definitions

SE 1.6 task · PDF p. 2, Activity 6.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

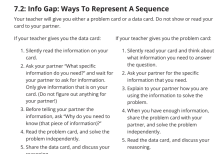
1.7 Representing More Sequences

Explore lesson 1.7

Standards connection

Students request specific information, explain why it is needed, solve independently after gathering it and discuss reasoning. HSF.BF.A.2 / HSF.LE.A.2: paired problem and data cards connect information gathering with sequence representations.

SE 1.7 task · PDF p. 2, Activity7.2 Info Gap directions


7.2: Info Gap: Ways To Represent A Sequence

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the data card:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need to know (that piece of information)?"
4. Read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

If your teacher gives you the problem card:

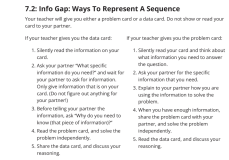
1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

SE 1.7 task · PDF p. 2, Activity 7.2

Conceptual understanding

The info-gap cards require students to move among a recurrence, a table and a graph. The recovered original cards include both a constant-factor recurrence and another rule that subtracts a changing term number, so not every case is geometric or arithmetic.

SE 1.7 task · PDF p. 2, Activity7.2 Info Gap directions


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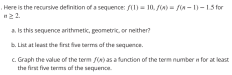
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3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

SE 1.7 task · PDF p. 2, Activity 7.2

Procedural skill and practice

Practice #1 asks students to classify a given recurrence, list terms and graph term value against position. The graph must represent the sequence inputs rather than a continuous curve by default.

SE 1.7 practice · PDF p. 1, Practice #1


1. Here is the recursive definition of a sequence: $f(1) = 10, f(n) = f(n-1) - 1.5$ for $n \geq 2$.

a. Is this sequence arithmetic, geometric, or neither?

b. List at least the first five terms of the sequence.

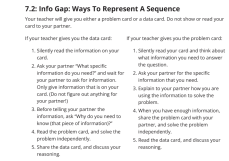
c. Graph the value of the term $f(n)$ as a function of the term number n for at least the first five terms of the sequence.

SE 1.7 practice · PDF p. 1, #1

Coherence: horizontal alignment

Earlier representation work gives students language for communicating a sequence. The information gap now requires them to request enough data to reconstruct it, making the sufficiency of those representations consequential.

SE 1.7 task · PDF p. 2, Activity7.2 Info Gap directions; 1.6 task · PDF p. 1, Warm-up 6.1


7.2: Info Gap: Ways To Represent A Sequence

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4. Read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

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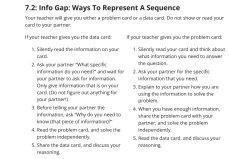
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4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

SE 1.7 task · PDF p. 2, Activity 7.2

Depth of reasoning

Students decide which information they need before requesting it and explain how they use each answer. The routine makes the sufficiency of an initial value and a rule part of the mathematical problem.

SE 1.7 task · PDF p. 2, Activity7.2 Info Gap directions; TE Student task, anticipated reasoning and Activity Synthesis


7.2: Info Gap: Ways To Represent A Sequence

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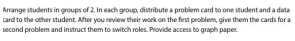
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3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

SE 1.7 task · PDF p. 2, Activity 7.2

Teacher supports

The teacher demonstrates one card set, reviews the first partner solution and then distributes another set with roles exchanged. This assigns each learner a mathematical contribution rather than treating group seating as participation.

TE 7.2 Launch; SE 1.7 task · PDF p. 2, Activity7.2 Info Gap directions

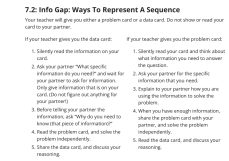

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles. Provide access to graph paper.

TE 7.2 Launch

Mathematical practices

MP6 is supported when students request specific information and explain why it is needed. The demand is precise mathematical communication, not only producing a final list of terms.

SE 1.7 task · PDF p. 2, Activity7.2 Info Gap directions; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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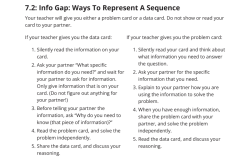
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4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

SE 1.7 task · PDF p. 2, Activity 7.2

Formative assessment

The explicit checkpoint after the first card problem lets the teacher inspect the representation before students switch roles. The later discussion connects the same initial value across notation, table and graph.

SE 1.7 task · PDF p. 2, Activity7.2 Info Gap directions; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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SE 1.7 task · PDF p. 2, Activity 7.2

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Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

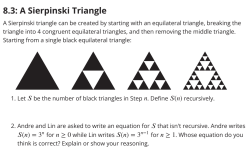
1.8 The nth Term

Explore lesson 1.8

Standards connection

Students connect recursive and explicit Sierpinski counts and interpret two different starting-index conventions. Both explicit rules can describe the same pictured list under their stated domains; original teacher guidance confirms this. HSF.BF.A.2: translating representations requires consistent initial-index assumptions.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2


8.3: A Sierpinski Triangle

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:

1. Let S be the number of black triangles in Step n. Define $S(n)$ recursively.

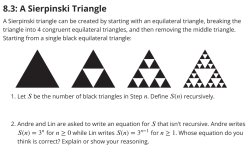
2. Andre and Lin are asked to write an equation for S that isn't recursive. Andre writes $S(n) = 3^n$ for $n \geq 0$ while Lin writes $S(n) = 3^{n-1}$ for $n \geq 1$. Whose equation do you think is correct? Explain or show your reasoning.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2

Conceptual understanding

The paper-cutting tasks connect repeated change with explicit rules, and the Sierpinski task connects those rules with initial-index choices. Both displayed Sierpinski formulas can describe the same list under their respective starting conventions.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2


8.3: A Sierpinski Triangle

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:

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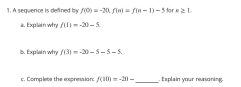
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SE 1.8 task · PDF p. 3, Activity8.3 #1–2

Procedural skill and practice

Practice #1 explains an explicitly defined arithmetic sequence by expanding its repeated subtraction. Students complete a later-term expression and justify the number of subtractions, rather than only calculate an answer.

SE 1.8 practice · PDF p. 1, Practice #1


1. A sequence is defined by $f(0) = -20$, $f(n) = f(n-1) - 5$ for $n \geq 1$.

a. Explain why $f(1) = -20 - 5$.

b. Explain why $f(3) = -20 - 5 - 5 - 5$.

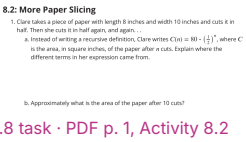
c. Complete the expression: $f(10) = -20 -$ _____. Explain your reasoning.

SE 1.8 practice · PDF p. 1, #1

Contextual application

The paper task connects geometric and arithmetic changes in area with explicit rules. Index choices must agree with the physical starting state; a correct expression with a different stated index is not an error.

SE 1.8 task · PDF p. 1, Activity 8.2; p. 2, Activity 8.2 #2


8.2: More Paper Slicing

1. Clare takes a piece of paper with length 8 inches and width 10 inches and cuts it in half. Then she cuts it in half again, and again, . . .

a. Instead of writing a recursive definition, Clare writes $C(n) = 80 \cdot \left(\frac{1}{2}\right)^n$, where C is the area, in square inches, of the paper after n cuts. Explain where the different terms in her expression came from.

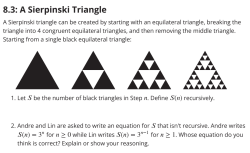
b. Approximately what is the area of the paper after 10 cuts?

SE 1.8 task · PDF p. 1, Activity 8.2

Coherence: horizontal alignment

The paper-folding situation returns from Lesson 2 with a demand for an explicit rule. Comparing different starting indices shows why two formulas can describe the same sequence when their input conventions differ.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2; 1.2 task · PDF p. 1, Activity 2.2


8.3: A Sierpinski Triangle

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:

1. Let S be the number of black triangles in Step n. Define $S(n)$ recursively.

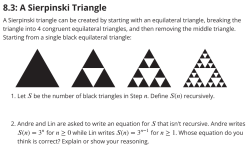
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SE 1.8 task · PDF p. 3, Activity8.3 #1–2

Depth of reasoning

In Activity 8.3, students compare two valid formulas whose first inputs differ. A sound explanation states what the index means; assuming that one student must be wrong misses the intended domain discussion.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2; TE Student task, anticipated reasoning and Activity Synthesis


8.3: A Sierpinski Triangle

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:

1. Let S be the number of black triangles in Step n. Define $S(n)$ recursively.

2. Andre and Lin are asked to write an equation for S that isn't recursive. Andre writes $S(n) = 3^n$ for $n \geq 0$ while Lin writes $S(n) = 3^{n-1}$ for $n \geq 1$. Whose equation do you think is correct? Explain or show your reasoning.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2

Teacher supports

The teacher explicitly anticipates the belief that different formulas cannot represent the same sequence. Students are encouraged to make a table beginning at each definition's stated first input and compare the generated lists.

TE 8.3 Anticipated Misconceptions and Activity Synthesis; SE 1.8 task · PDF p. 3, Activity8.3 #1–2


Anticipated Misconceptions

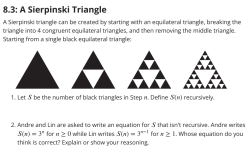
Students may assume that at least one person has to be wrong because their equations aren't equivalent. Encourage them to make a table for each equation that starts with the first value for n according to the definitions.

TE 8.3 Anticipated Misconceptions

Mathematical practices

MP6 is supported by explaining the meaning of the starting index and using notation consistently. A changed exponent is meaningful only together with the corresponding domain and initial term.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


8.3: A Sierpinski Triangle

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:

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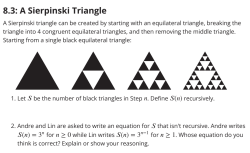
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SE 1.8 task · PDF p. 3, Activity8.3 #1–2

Formative assessment

The teacher selects explanations with clear index assumptions and uses the discussion to distinguish equivalent lists from identical input-output conventions. This checks a specific source of off-by-one reasoning before new contexts are introduced.

SE 1.8 task · PDF p. 3, Activity8.3 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


8.3: A Sierpinski Triangle

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:

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SE 1.8 task · PDF p. 3, Activity8.3 #1–2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

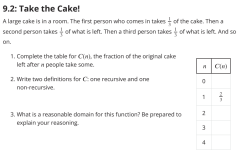
1.9 What's the Equation?

Explore lesson 1.9

Standards connection

Students tabulate remaining cake, write both recursive and nonrecursive definitions and explain a reasonable domain. Form selection and domain interpretation are required, beyond calculating successive fractions. HSF.BF.A.2 and HSF.IF.B.5: model a geometric situation in both forms and relate permitted inputs to people taking cake.

SE 1.9 task · PDF p. 1, Activity9.2 #1–3


9.2: Take the Cake!

A large cake is in a room. The first person who comes in takes $\frac{1}{2}$ of the cake. Then a second person takes $\frac{1}{2}$ of what is left. Then a third person takes $\frac{1}{2}$ of what is left. And so on.

1. Complete the table for $C(n)$, the fraction of the original cake left after n people take some.
2. Write two definitions for C: one recursive and one non-recursive.
3. What is a reasonable domain for this function? Be prepared to explain your reasoning.

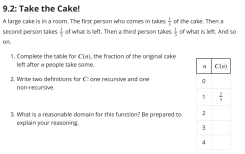
n	$C(n)$
0	
1	$\frac{1}{2}$
2	
3	
4	

SE 1.9 task · PDF p. 1, Activity 9.2

Conceptual understanding

The cake task requires a recursive and an explicit rule for the remaining amount and a reasonable domain. The Fibonacci task then shows that a recurrence can be straightforward even when a convenient explicit rule is not.

SE 1.9 task · PDF p. 1, Activity9.2 #1–3


9.2: Take the Cake!

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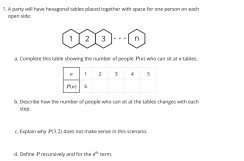
n	$C(n)$
0	
1	$\frac{1}{2}$
2	
3	
4	

SE 1.9 task · PDF p. 1, Activity 9.2

Procedural skill and practice

Practice #1 develops the seating count for joined hexagonal tables, then writes both forms of its rule. Rejecting a fractional number of tables makes domain interpretation part of the practice.

SE 1.9 practice · PDF p. 1, Practice #1


1. A party will have hexagonal tables placed together with space for one person on each open side:

a. Complete this table showing the number of people $P(n)$ who can sit at n tables.

n	1	2	3	4	5
$P(n)$	6				

b. Describe how the number of people who can sit at the tables changes with each step.

c. Explain why $P(3.2)$ does not make sense in this scenario.

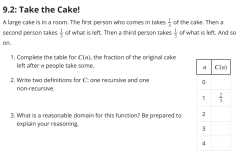
d. Define P recursively and for the n^{th} term.

SE 1.9 practice · PDF p. 1, #1

Contextual application

Students describe cake remaining after repeated servings with recursive and explicit rules. The initial amount, serving pattern and nonnegative integer input domain give the expressions their contextual meaning.

SE 1.9 task · PDF p. 1, Activity 9.2


9.2: Take the Cake!

A large cake is in a room. The first person who comes in takes $\frac{1}{2}$ of the cake. Then a second person takes $\frac{1}{2}$ of what is left. Then a third person takes $\frac{1}{2}$ of what is left. And so on.

1. Complete the table for $C(n)$, the fraction of the original cake left after n people take some.
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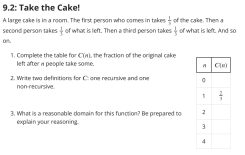
n	$C(n)$
0	
1	$\frac{1}{2}$
2	
3	
4	

SE 1.9 task · PDF p. 1, Activity 9.2

Coherence: horizontal alignment

The preceding explicit-rule work is used to describe cake remaining after repeated servings. Students must now interpret the starting value and input domain in a context, not merely match a list.

SE 1.9 task · PDF p. 1, Activity9.2 #1–3; 1.8 task · PDF p. 1, Activity 8.2


9.2: Take the Cake!

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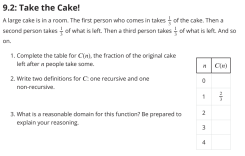
n	$C(n)$
0	
1	$\frac{1}{2}$
2	
3	
4	

SE 1.9 task · PDF p. 1, Activity 9.2

Depth of reasoning

Students must decide which inputs make sense for people taking cake and explain how their two rules represent the same process. Algebraically extending a formula does not automatically make every input meaningful in the situation.

SE 1.9 task · PDF p. 1, Activity9.2 #1–3; TE Student task, anticipated reasoning and Activity Synthesis


9.2: Take the Cake!

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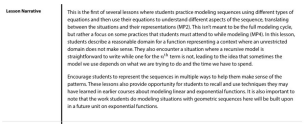
n	$C(n)$
0	
1	$\frac{1}{2}$
2	
3	
4	

SE 1.9 task · PDF p. 1, Activity 9.2

Teacher supports

The preparation explicitly frames this lesson as selected modeling practices rather than a full modeling cycle. Teacher guidance focuses attention on domain and translates the situation into the two sequence forms.

TE Lesson Narrative; SE 1.9 task · PDF p. 1, Activity9.2 #1–3


Lesson Narrative

This is the first of several lessons where students practice modeling sequences using different types of equations and then use their equations to understand different aspects of the sequence, translating between the situations and their representations (MP2). This isn't meant to be the full modeling cycle, but rather a focus on some practices that students must attend to while modeling (MP4). In this lesson, students describe a reasonable domain for a function representing a context where an unrestricted domain does not make sense. They also encounter a situation where a recursive model is straightforward to write while one for the n^{th} term is not, leading to the idea that sometimes the model we use depends on what we are trying to do and the time we have to spend.

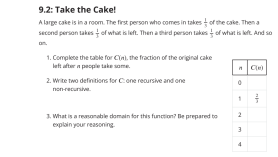
Encourage students to represent the sequences in multiple ways to help them make sense of the patterns. These lessons also provide opportunity for students to recall and use techniques they may have learned in earlier courses about modeling linear and exponential functions. It is also important to note that the work students do modeling situations with geometric sequences here will be built upon in a future unit on exponential functions.

TE Lesson Narrative

Mathematical practices

MP4 is supported in a limited way when students represent the cake process and evaluate a sensible domain. The quantities and process are supplied, so independent problem formulation is not established.

SE 1.9 task · PDF p. 1, Activity9.2 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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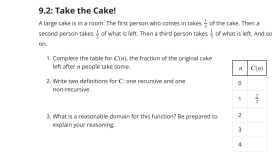
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1	$\frac{1}{2}$
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3	
4	

SE 1.9 task · PDF p. 1, Activity 9.2

Formative assessment

The teacher pauses after the first Fibonacci construction to check that students interpreted the drawing instructions correctly. Later predictions can then reveal recursive reasoning rather than an unnoticed drawing error.

SE 1.9 task · PDF p. 1, Activity9.2 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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A large cake is in a room. The first person who comes in takes $\frac{1}{2}$ of the cake. Then a second person takes $\frac{1}{2}$ of what is left. Then a third person takes $\frac{1}{3}$ of what is left. And so on.

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n	$C(n)$
0	
1	$\frac{1}{2}$
2	
3	
4	

SE 1.9 task · PDF p. 1, Activity 9.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

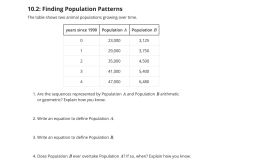
1.10 Situations and Sequence Types

Explore lesson 1.10

Standards connection

Students classify two population tables, construct equations and determine whether and when one population overtakes the other. Model type is selected from data, although continuation of the pattern is assumed. HSF.LE.A.2/HSF.BF.A.2: construct linear/exponential sequence functions from tables and interpret their predictions.

SE 1.10 task · PDF p. 1, Activity10.2 #1–4


10.2: Finding Population Patterns

The table shows two animal populations growing over time.

years since 1998	Population A	Population B
0	23,080	3,125
1	29,890	3,750
2	35,860	4,500
3	41,890	5,400
4	47,980	6,480

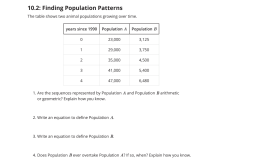
1. Are the sequences represented by Population A and Population B arithmetic or geometric? Explain how you know.
2. Write an equation to define Population A.
3. Write an equation to define Population B.
4. Does Population B ever overtake Population A? If so, when? Explain how you know.

SE 1.10 task · PDF p. 1, Activity 10.2

Conceptual understanding

Students distinguish additive and multiplicative population change from a table and choose equations that fit. Comparing the projections requires interpreting the resulting functions, not only noticing a pattern in five rows.

SE 1.10 task · PDF p. 1, Activity10.2 #1–4


10.2: Finding Population Patterns

The table shows two animal populations growing over time.

years since 1998	Population A	Population B
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1	29,890	3,750
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4	47,980	6,480

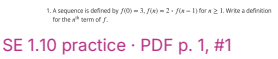
1. Are the sequences represented by Population A and Population B arithmetic or geometric? Explain how you know.
2. Write an equation to define Population A.
3. Write an equation to define Population B.
4. Does Population B ever overtake Population A? If so, when? Explain how you know.

SE 1.10 task · PDF p. 1, Activity 10.2

Procedural skill and practice

Practice #1 converts an arithmetic recurrence into a rule for a specified term. Students must preserve the starting index while replacing repeated addition with a direct calculation.

SE 1.10 practice · PDF p. 1, Practice #1

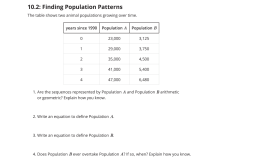

1. A sequence is defined by $f(0) = 3$, $f(n) = 2 \cdot f(n-1)$ for $n \geq 1$. Write a definition for the n^{th} term of f.

SE 1.10 practice · PDF p. 1, #1

Contextual application

Population changes require students to decide whether repeated addition or multiplication fits the stated process. The resulting rule depends on maintaining the supplied rate assumption beyond the listed data.

SE 1.10 task · PDF p. 1, Activity 10.2


10.2: Finding Population Patterns

The table shows two animal populations growing over time.

years since 1998	Population A	Population B
0	23,080	3,125
1	29,890	3,750
2	35,860	4,500
3	41,890	5,400
4	47,980	6,480

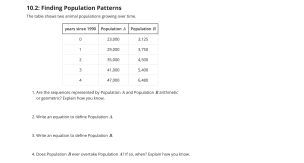
1. Are the sequences represented by Population A and Population B arithmetic or geometric? Explain how you know.
2. Write an equation to define Population A.
3. Write an equation to define Population B.
4. Does Population B ever overtake Population A? If so, when? Explain how you know.

SE 1.10 task · PDF p. 1, Activity 10.2

Coherence: horizontal alignment

Lesson 9 supplies a setting for recursive and explicit descriptions; this lesson leaves more of the form selection to students. Choosing a representation becomes part of solving the population problem.

SE 1.10 task · PDF p. 1, Activity10.2 #1–4; 1.9 task · PDF p. 1, Activity 9.2

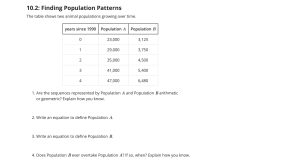


SE 1.10 task · PDF p. 1, Activity 10.2

Depth of reasoning

The population task asks whether and when one population overtakes the other and requires an explanation. A useful representation must support the comparison beyond the displayed rows while making the continuation assumption clear.

SE 1.10 task · PDF p. 1, Activity10.2 #1–4; TE Student task, anticipated reasoning and Activity Synthesis

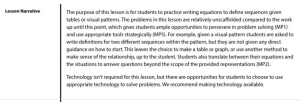


SE 1.10 task · PDF p. 1, Activity 10.2

Teacher supports

The preparation intentionally describes these tasks as relatively unscaffolded. Students choose how to begin, while the teacher can prompt earlier representations if a learner cannot identify the sequence type.

TE Lesson Narrative; SE 1.10 task · PDF p. 1, Activity10.2 #1–4

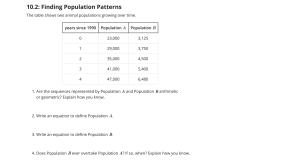


TE Lesson Narrative

Mathematical practices

MP1 is supported by the need to choose a representation and pursue the comparison with little initial procedural direction. Making technology available is an affordance, not evidence that every student explains a tool choice.

SE 1.10 task · PDF p. 1, Activity10.2 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

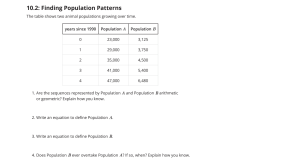


SE 1.10 task · PDF p. 1, Activity 10.2

Formative assessment

The teacher compares how students used tables, equations and graphs to answer the overtaking question. If a selected rule does not regenerate the table, that check exposes an inappropriate model before interpreting its forecast.

SE 1.10 task · PDF p. 1, Activity10.2 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 1.10 task · PDF p. 1, Activity 10.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

1.11 Adding Up

Explore lesson 1.11

Standards connection

Students distinguish the snowflake side-count sequence, newly added triangle counts and cumulative triangle total. These are related quantities with different rules. HSF.BF.A.2 contribution and preparation for HSA.SSE.B.4; finite-sum formula derivation occurs later, not here.

SE 1.11 task · PDF p. 2, Activity11.3 #1–3



SE 1.11 task · PDF p. 2, Activity 11.3

Conceptual understanding

Students distinguish the terms of a sequence from the accumulated sum of those terms. The snowflake separates side counts, newly added triangles and total triangles, which are related but different quantities.

SE 1.11 task · PDF p. 2, Activity11.3 #1–3

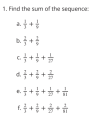


SE 1.11 task · PDF p. 2, Activity 11.3

Procedural skill and practice

Practice #1 adds finite lists of fractions with repeated factors. This practices the arithmetic of partial sums; the general finite geometric-sum formula is developed in Unit 2.

SE 1.11 practice · PDF p. 1, Practice #1


1. Find the sum of the sequence:

a. $\frac{1}{3}+\frac{1}{3}$

b. $\frac{1}{3}+\frac{1}{3}$

c. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

d. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

e. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

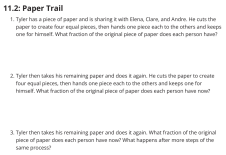
f. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

SE 1.11 practice · PDF p. 1, #1

Contextual application

The gift investigation distinguishes the number given on one day from the total received. Students must accumulate the relevant terms rather than report the final daily amount.

SE 1.11 task · PDF p. 1, Activity 11.2


11.2: Paper Trail

1. Tyler has a piece of paper and is sharing it with Elena, Clare, and Andre. He cuts the paper to create four equal pieces, then hands one piece each to the others and keeps one for himself. What fraction of the original piece of paper does each person have?

2. Tyler then takes his remaining paper and does it again. He cuts the paper to create four equal pieces, then hands one piece each to the others and keeps one for himself. What fraction of the original piece of paper does each person have now?

3. Tyler then takes his remaining paper and does it again. What fraction of the original piece of paper does each person have now? What happens after more steps of the same process?

SE 1.11 task · PDF p. 1, Activity11.2 #1–3

Coherence: horizontal alignment

The cake problem tracks the amount remaining at one time; the gifts problem asks for an accumulated total. Distinguishing a sequence term from a sum prevents using an appropriate term rule to answer the wrong question.

SE 1.11 task · PDF p. 2, Activity11.3 #1–3; 1.9 task · PDF p. 1, Activity 9.2


11.3: A Threefold Design

SE 1.11 task · PDF p. 2, Activity 11.3

Depth of reasoning

The snowflake questions require students to explain newly added triangles before computing a total. Summing the side-count sequence instead would answer a different question, so the quantity being accumulated must be justified.

SE 1.11 task · PDF p. 2, Activity11.3 #1–3; TE Student task, anticipated reasoning and Activity Synthesis

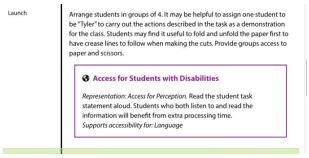

11.3: A Threefold Design

SE 1.11 task · PDF p. 2, Activity 11.3

Teacher supports

The Paper Trail launch offers physical enactment, read-aloud and additional processing time. These supports clarify who keeps or receives each piece while preserving the cumulative-quantity reasoning.

TE 11.2 Launch; SE 1.11 task · PDF p. 2, Activity11.3 #1–3


Launch

Arrange students in groups of 4. It may be helpful to assign one student to be "Tyler" to carry out the actions described in the task as a demonstration for the class. Students may find it useful to fold and unfold the paper first to have crease lines to follow when making the cuts. Provide groups access to paper and scissors.

Access for Students with Disabilities

Representation: Access for Perception. Read the student task statement aloud. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language

TE 11.2 Launch

Mathematical practices

MP2 is supported by deciding which numerical sequence represents sides, new triangles or accumulated pieces. The meaning of a term determines whether summing it answers the stated question.

SE 1.11 task · PDF p. 2, Activity11.3 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


11.3: A Threefold Design

SE 1.11 task · PDF p. 2, Activity 11.3

Formative assessment

The teacher listens for whether students distinguish Tyler's shrinking remainder from the other learners' growing totals. Contrasting these quantities helps diagnose a model-interpretation error rather than only a fraction-calculation error.

SE 1.11 task · PDF p. 2, Activity11.3 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


11.3: A Threefold Design

SE 1.11 task · PDF p. 2, Activity 11.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

2.1 Let's Make a Box

Explore lesson 2.1

Standards connection

Students build and measure an open box, make a plan for greatest volume, pause for plan review and write a volume expression. Physical positivity restricts valid cutout sizes. HSF.BF.A.1a/HSA.CED.A.2/HSF.IF.B.5: formulate a function from physical dimensions and connect feasible domain to the model.

SE 2.1 task · PDF p. 2, Activity1.2 #1–2; Activity1.3 #1–2, graph continuation PDFp3


1.3: Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

Pause here so your teacher can review your plan.

2. Write an expression for $V(x)$.

SE 2.1 task · PDF p. 2, Activity 1.3

Conceptual understanding

Cutting a single square size from every corner changes all three box dimensions. Students connect a physical construction and measured volumes with a polynomial volume expression and a graph-based maximum.

SE 2.1 task · PDF p. 2, Activity1.2 #1–2; Activity1.3 #1–2, graph continuation PDFp3


1.3: Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

Pause here so your teacher can review your plan.

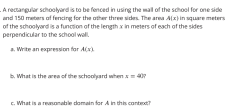
2. Write an expression for $V(x)$.

SE 2.1 task · PDF p. 2, Activity 1.3

Procedural skill and practice

Practice #1 writes an area expression for a schoolyard bounded by a wall and a fixed amount of fencing, evaluates it and gives a reasonable domain. The practice transfers the quantity-and-constraint reasoning from boxes to another shape.

SE 2.1 practice · PDF p. 1, Practice #1


1. A rectangular schoolyard is to be fenced in using the wall of the school for one side and 150 meters of fencing for the other three sides. The area $A(x)$ in square meters of the schoolyard is a function of the length x in meters of each of the sides perpendicular to the school wall.

a. Write an expression for $A(x)$.

b. What is the area of the schoolyard when $x = 40$?

c. What is a reasonable domain for A in this context?

SE 2.1 practice · PDF p. 1, #1

Contextual application

Students construct an open box and relate cut size to dimensions and volume. Maximizing the volume requires a domain consistent with a real box, not every input accepted by the polynomial.

SE 2.1 task · PDF p. 2, Activities 1.2–1.3


1.3: Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

Pause here so your teacher can review your plan.

2. Write an expression for $V(x)$.

SE 2.1 task · PDF p. 2, Activity 1.3

Coherence: horizontal alignment

The measured box table precedes the general volume expression. Moving between the physical construction, permissible cut sizes and the graph makes the maximum a property of the constrained model.

SE 2.1 task · PDF p. 2, Activity1.2 #1–2; Activity1.3 #1–2, graph continuation PDFp3; p. 2, Activities 1.2–1.3


1.3: Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

Pause here so your teacher can review your plan.

2. Write an expression for $V(x)$.

SE 2.1 task · PDF p. 2, Activity 1.3

Depth of reasoning

The main task asks students to make an optimization plan before writing the expression and graphing. A proposed maximum must correspond to positive, constructible dimensions, not any input allowed by the polynomial.

SE 2.1 task · PDF p. 2, Activity1.2 #1–2; Activity1.3 #1–2, graph continuation PDFp3; TE Student task, anticipated reasoning and Activity Synthesis


1.3: Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

Pause here so your teacher can review your plan.

2. Write an expression for $V(x)$.

SE 2.1 task · PDF p. 2, Activity 1.3

Teacher supports

The preparation specifies rulers, scissors, paper and tape for pairs and access to graphing technology. These supplies let students test the physical dimensions before generalizing the volume relationship.

TE Required Preparation; SE 2.1 task · PDF p. 2, Activity1.2 #1–2; Activity1.3 #1–2, graph continuation PDFp3


Required Preparation

Have rulers, scissors, sheets of paper, and tape available for groups of 2 to use to make their boxes. Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if each student has their own device. (Desmos is available under Math Tools.)

TE Required Preparation

Mathematical practices

MP4 is supported when students formulate a volume expression from the construction and interpret an approximate maximum. The paper and cutout construction are supplied, so the problem situation itself is not independently selected.

SE 2.1 task · PDF p. 2, Activity1.2 #1–2; Activity1.3 #1–2, graph continuation PDFp3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


1.3: Building the Biggest Box

SE 2.1 task · PDF p. 2, Activity 1.3

Formative assessment

The task pauses for a teacher review of each optimization plan before the graphing step. Guidance also checks whether learners subtracted the cutout length from both ends of each paper dimension.

SE 2.1 task · PDF p. 2, Activity1.2 #1–2; Activity1.3 #1–2, graph continuation PDFp3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


1.3: Building the Biggest Box

SE 2.1 task · PDF p. 2, Activity 1.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

2.2 Funding the Future

Explore lesson 2.2

Standards connection

Students create accumulated-value expressions as deposits at different times are added, then use the final graph to estimate interest rate. Growth-factor variable and accumulation convention are supplied. HSA.SSE.A.1/HSA.CED.A.2: interpret terms as differently timed deposits and represent their combined value; not independent variable selection.

SE 2.2 task · PDF p. 3, Activity2.3 #1–4


2.3: A Yearly Gift

SE 2.2 task · PDF p. 3, Activity 2.3

Conceptual understanding

The investment model assigns different powers of the growth factor to deposits made at different times. Students connect those polynomial terms with the balance and its graph; the growth-factor variable is supplied by the task.

SE 2.2 task · PDF p. 3, Activity2.3 #1–4

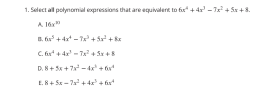

2.3: A Yearly Gift

SE 2.2 task · PDF p. 3, Activity 2.3

Procedural skill and practice

Practice #1 selects polynomial expressions equivalent to a given expression. Reordered terms can preserve value, but combining unlike powers would not.

SE 2.2 practice · PDF p. 1, Practice #1


1. Select all polynomial expressions that are equivalent to $6x^4 + 4x^3 - 7x^2 + 5x + 8$.

SE 2.2 practice · PDF p. 1, #1

Contextual application

The investment expression assigns different growth periods to deposits made at different times. Students must explain the terms before comparing account values; the supplied fixed rate is a modeling assumption.

SE 2.2 task · PDF p. 3, Activity 2.3


2.3: A Yearly Gift

SE 2.2 task · PDF p. 3, Activity 2.3

Coherence: horizontal alignment

The timed investment deposits introduce a polynomial expression for accumulated value. Lesson 26 returns to regular contributions with a finite geometric sum; the timing of each contribution determines its growth period in both tasks.

SE 2.2 task · PDF p. 3, Activity2.3 #1–4; 2.26 task · PDF p. 3, Activity 26.3


2.3: A Yearly Gift

SE 2.2 task · PDF p. 3, Activity 2.3

Depth of reasoning

Students revise the accumulated-value expression as additional deposits are introduced and then infer an interest rate from a target balance. Their explanation must account for how long each deposit earns interest, rather than treat every contribution alike.

SE 2.2 task · PDF p. 3, Activity2.3 #1–4; TE Student task, anticipated reasoning and Activity Synthesis

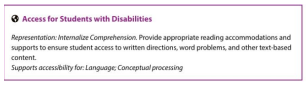


SE 2.2 task · PDF p. 3, Activity 2.3

Teacher supports

The investment launch provides reading accommodations so students can access the word problem while still formulating its polynomial. This addresses text processing; it does not turn the supplied growth-factor variable into a student choice.

TE 2.3 Launch; Representation reading support; SE 2.2 task · PDF p. 3, Activity2.3 #1–4



TE 2.3 Launch; Representation reading support

Mathematical practices

MP2 is supported by interpreting a polynomial's coefficients and powers as deposits and accumulation times. Correct symbolic expansion alone would not explain those quantities.

SE 2.2 task · PDF p. 3, Activity2.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.2 task · PDF p. 3, Activity 2.3

Formative assessment

The teacher pauses after the first deposit cases and compares how students represented the growing balance. The discussion can reveal whether an exponent represents elapsed years for the correct deposit before the complete expression is assembled.

SE 2.2 task · PDF p. 3, Activity2.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.2 task · PDF p. 3, Activity 2.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

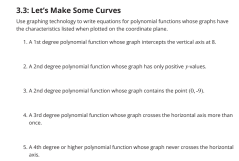
2.3 Introducing Polynomials

Explore lesson 2.3

Standards connection

Students author polynomial functions with specified degree, intercept or positivity properties using graphing technology. This develops graph profiles and feature constraints. HSF.IF.C.7 contribution. This selected task does not by itself establish HSA.SSE.A.2 because useful rewriting through structure is not required.

SE 2.3 task · PDF p. 1, Activity3.3 #1–5

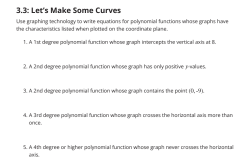


SE 2.3 task · PDF p. 1, Activity 3.3

Conceptual understanding

Students build a profile of polynomial functions through degree, intercepts, relative extrema and graphs. Writing an equation with specified features asks for more than naming the degree of a supplied expression.

SE 2.3 task · PDF p. 1, Activity3.3 #1–5

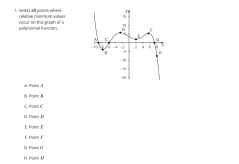


SE 2.3 task · PDF p. 1, Activity 3.3

Procedural skill and practice

Practice #1 identifies relative minima from a labeled graph. Learners must compare a point with nearby graph values, not simply choose the lowest point shown.

SE 2.3 practice · PDF p. 1, Practice #1

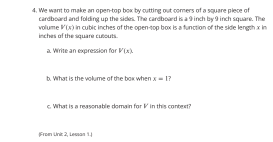


SE 2.3 practice · PDF p. 1, #1

Contextual application

The box expression must be interpreted through the lengths produced by cutting and folding. The polynomial can be evaluated outside the physical domain, so algebraic existence alone does not make a box possible.

SE 2.3 practice · PDF p. 2, Practice #4


4. We want to make an open-top box by cutting out corners of a square piece of cardboard and folding up the sides. The cardboard is a 9 inch by 9 inch square. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts.

a. Write an expression for $V(x)$.

b. What is the volume of the box when $x = 1$?

c. What is a reasonable domain for V in this context?

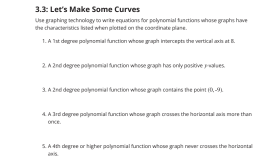
(From Unit 2, Lesson 1.)

SE 2.3 practice · PDF p. 2, Practice #4

Coherence: horizontal alignment

The box volume expression supplies a prior cubic example for the new polynomial vocabulary. Rewriting into standard form connects the construction to degree and leading coefficient rather than treating the definition as an isolated list of terms.

SE 2.3 task · PDF p. 1, Activity3.3 #1–5; 2.1 task · PDF p. 2, Activities 1.2–1.3


3.3: Let's Make Some Curves

Use graphing technology to write equations for polynomial functions whose graphs have the characteristics listed when plotted on the coordinate plane.

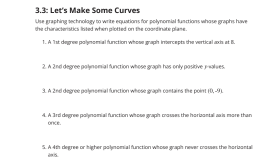
1. A 1st degree polynomial function whose graph intercepts the vertical axis at 8.
2. A 2nd degree polynomial function whose graph has only positive y-values.
3. A 2nd degree polynomial function whose graph contains the point $(0, -9)$.
4. A 3rd degree polynomial function whose graph crosses the horizontal axis more than once.
5. A 4th degree or higher polynomial function whose graph never crosses the horizontal axis.

SE 2.3 task · PDF p. 1, Activity 3.3

Depth of reasoning

The curve-making task asks students to produce functions that meet degree and sign or intercept conditions and explain their choices. It develops graph-feature reasoning but does not by itself require choosing a structural rewrite under SSE.A.2.

SE 2.3 task · PDF p. 1, Activity3.3 #1–5; TE Student task, anticipated reasoning and Activity Synthesis


3.3: Let's Make Some Curves

Use graphing technology to write equations for polynomial functions whose graphs have the characteristics listed when plotted on the coordinate plane.

1. A 1st degree polynomial function whose graph intercepts the vertical axis at 8.
2. A 2nd degree polynomial function whose graph has only positive y-values.
3. A 2nd degree polynomial function whose graph contains the point $(0, -9)$.
4. A 3rd degree polynomial function whose graph crosses the horizontal axis more than once.
5. A 4th degree or higher polynomial function whose graph never crosses the horizontal axis.

SE 2.3 task · PDF p. 1, Activity 3.3

Teacher supports

The preparation separates introductory graph exploration from factor-and-zero connections developed later. Teacher support suggests testing an input or using an equation frame when students cannot begin, while retaining the feature constraints.

TE Lesson Narrative; SE 2.3 task · PDF p. 1, Activity3.3 #1–5


Lesson Narrative

The purpose of this lesson is to formally define what the expression for a polynomial function can look like when written in standard form and to expose students to a wide variety of graphs of polynomials. Students are introduced to the new terms **degree, relative minimum,** and **relative maximum** for describing features of expressions and graphs. The degree of a polynomial is the highest exponent occurring on x when the polynomial is written out as a sum of non-zero constants times powers of x, with like terms collected. The relative minimum is a point on the graph of a function that is lower than any of the points around it, and the relative maximum is a point on the graph of a function that is higher than any of the points around it.

Students are also reminded of other useful vocabulary for polynomials, such as leading term and constant term, and use precise language to describe expressions and identify polynomials (MP6). In this lesson, students use vocabulary to make connections between polynomials and their graphs. They will continue to deepen their understanding of these connections throughout the unit.

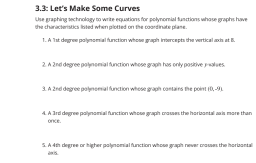
In the second activity, students match equations to graphs by focusing on the structure (MP7). Students will continue this work in future lessons as they make connections between the structure of a polynomial written in factored form and the zeros of the function. In the final activity, students have the opportunity to play with graphing a variety of polynomials as they try to make graphs that meet specific criteria. This is meant to be informal and an opportunity for students to improve their skill graphing with technology while building their mental profile of what graphs of polynomials can look like.

TE Lesson Narrative

Mathematical practices

MP6 is supported by using degree, intercept and relative-extremum language to state and check a function's properties. An approximate graph needs to be interpreted with those definitions, not only by visual resemblance.

SE 2.3 task · PDF p. 1, Activity3.3 #1–5; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


3.3: Let's Make Some Curves

Use graphing technology to write equations for polynomial functions whose graphs have the characteristics listed when plotted on the coordinate plane.

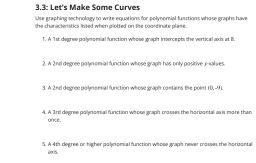
1. A 1st degree polynomial function whose graph intercepts the vertical axis at 8.
2. A 2nd degree polynomial function whose graph has only positive y-values.
3. A 2nd degree polynomial function whose graph contains the point $(0, -9)$.
4. A 3rd degree polynomial function whose graph crosses the horizontal axis more than once.
5. A 4th degree or higher polynomial function whose graph never crosses the horizontal axis.

SE 2.3 task · PDF p. 1, Activity 3.3

Formative assessment

The teacher collects students' remaining questions about polynomials on slips and displays them for discussion. These questions can guide follow-up on degree and graph features without claiming the protected cool-down has been inspected.

SE 2.3 task · PDF p. 1, Activity3.3 #1–5; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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SE 2.3 task · PDF p. 1, Activity 3.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

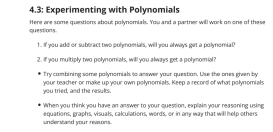
2.4 Combining Polynomials

Explore lesson 2.4

Standards connection

Students transfer closure questions from integers to polynomial addition/subtraction/multiplication, test examples and explain generally using chosen representations. HSA.APR.A.1: integer analogy, closure and operations; examples support conjecture but do not alone establish a general proof.

SE 2.4 task · PDF p. 2, Activity4.2 #7–9; Activity4.3 #1–2 and response directions


4.3: Experimenting with Polynomials

Here are some questions about polynomials. You and a partner will work on one of these questions.

1. If you add or subtract two polynomials, will you always get a polynomial?
2. If you multiply two polynomials, will you always get a polynomial?

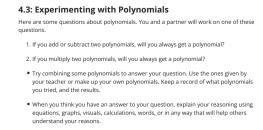
- Try combining some polynomials to answer your question. Use the ones given by your teacher or make up your own polynomials. Keep a record of what polynomials you tried, and the results.
- When you think you have an answer to your question, explain your reasoning using equations, graphs, visuals, calculations, words, or in any way that will help others understand your reasons.

SE 2.4 task · PDF p. 2, Activity 4.3

Conceptual understanding

The lesson compares closure of integer operations with closure of polynomial operations. Students distinguish testing examples from explaining why sums, differences and products remain in the polynomial system.

SE 2.4 task · PDF p. 2, Activity4.2 #7–9; Activity4.3 #1–2 and response directions


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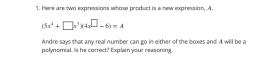
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SE 2.4 task · PDF p. 2, Activity 4.3

Procedural skill and practice

Practice #1 asks whether arbitrary real values in coefficient positions preserve a polynomial product. Learners must distinguish restrictions on exponents from the different role of coefficients.

SE 2.4 practice · PDF p. 1, Practice #1


1. Here are two expressions whose product is a new expression, A.

$(5x^4 + \square x^3)(4\square - 6) = A$

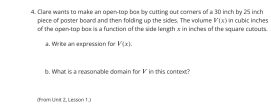
Andre says that any real number can go in either of the boxes and A will be a polynomial. Is he correct? Explain your reasoning.

SE 2.4 practice · PDF p. 1, #1

Contextual application

The box problem connects a polynomial expression with permissible cut sizes. Students must preserve positive dimensions when interpreting the expression, beyond simply performing polynomial operations.

SE 2.4 practice · PDF p. 1, Practice #4


4. Clare wants to make an open-top box by cutting out corners of a 30 inch by 25 inch piece of poster board and then folding up the sides. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts.

a. Write an expression for $V(x)$.

b. What is a reasonable domain for V in this context?

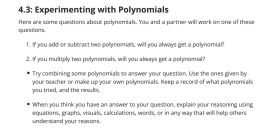
(From Unit 2, Lesson 1.)

SE 2.4 practice · PDF p. 1, Practice #4

Coherence: horizontal alignment

The polynomial definition established in Lesson 3 is used to judge results of operations. Explaining closure requires identifying why the resulting exponents and terms still meet that definition.

SE 2.4 task · PDF p. 2, Activity4.2 #7–9; Activity4.3 #1–2 and response directions; 2.3 task · PDF p. 2, Lesson Summary


4.3: Experimenting with Polynomials

Here are some questions about polynomials. You and a partner will work on one of these questions.

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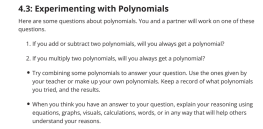
- Try combining some polynomials to answer your question. Use the ones given by your teacher or make up your own polynomials. Keep a record of what polynomials you tried, and the results.
- When you think you have an answer to your question, explain your reasoning using equations, graphs, visuals, calculations, words, or in any way that will help others understand your reasons.

SE 2.4 task · PDF p. 2, Activity 4.3

Depth of reasoning

Students justify a closure claim and examine purported counterexamples. A failed trial may come from a nonpolynomial input or an arithmetic error, so it is not automatically a counterexample to closure.

SE 2.4 task · PDF p. 2, Activity4.2 #7–9; Activity4.3 #1–2 and response directions; TE Student task, anticipated reasoning and Activity Synthesis


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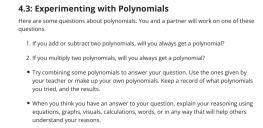
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SE 2.4 task · PDF p. 2, Activity 4.3

Teacher supports

The teacher offers a true/false organizer and sentence frames, then checks the causes of apparent counterexamples. The response preserves the general closure question rather than reducing it to repeated multiplication.

TE 4.3 Anticipated Misconceptions and Activity Synthesis; SE 2.4 task · PDF p. 2, Activity4.2 #7–9; Activity4.3 #1–2 and response directions


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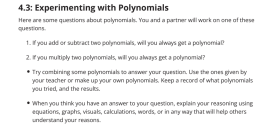
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- When you think you have an answer to your question, explain your reasoning using equations, graphs, visuals, calculations, words, or in any way that will help others understand your reasons.

SE 2.4 task · PDF p. 2, Activity 4.3

Mathematical practices

MP3 is supported by giving reasons for a universal operation claim and evaluating examples presented against it. A few confirming examples cannot alone prove closure.

SE 2.4 task · PDF p. 2, Activity4.2 #7–9; Activity4.3 #1–2 and response directions; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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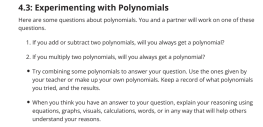
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- When you think you have an answer to your question, explain your reasoning using equations, graphs, visuals, calculations, words, or in any way that will help others understand your reasons.

SE 2.4 task · PDF p. 2, Activity 4.3

Formative assessment

As groups explain their experiments, the teacher checks whether both starting expressions are polynomials and whether the operations were performed correctly. That separates a definition error from a computational error before discussing the general claim.

SE 2.4 task · PDF p. 2, Activity4.2 #7–9; Activity4.3 #1–2 and response directions; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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SE 2.4 task · PDF p. 2, Activity 4.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

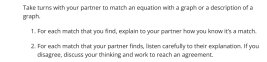
2.5 Connecting Factors and Zeros

Explore lesson 2.5

Standards connection

Students match factored polynomials to graphs/descriptions and justify each match while responding to disagreement. Zeros require the opposite sign of the linear-factor constant. HSA.APR.B.3: use factors to identify zeros and connect them to intercepts; separate card content limits complete match audit.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions; equation list on PDFp2

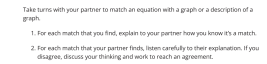

Take turns with your partner to match an equation with a graph or a description of a graph.
1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions #1–2

Conceptual understanding

Students connect a zero factor with a zero product and the corresponding horizontal intercept. Matching an equation to a graph requires using more than one feature when two graphs have similar intercepts.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions; equation list on PDFp2

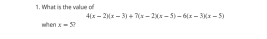

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2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions #1–2

Procedural skill and practice

Practice #1 evaluates a structured expression at a value that makes a repeated factor zero. Recognizing the zero factor can avoid unnecessary expansion.

SE 2.5 practice · PDF p. 1, Practice #1

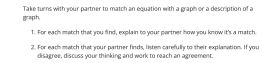

1. What is the value of $4(x-2)(x-5)+7(x-2)(x-5)-6(x-5)(x-5)$ when $x=5$?

SE 2.5 practice · PDF p. 1, #1

Coherence: horizontal alignment

Earlier polynomial examples are classified by algebraic structure. This lesson adds a graph connection: a factored expression identifies zeros, and students must distinguish zeros from other intercept information.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions; equation list on PDFp2; 2.3 task · PDF p. 1, Activity 3.3

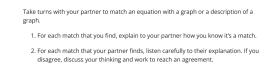

Take turns with your partner to match an equation with a graph or a description of a graph.
1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions #1–2

Depth of reasoning

In the card sort, partners explain matches and resolve disagreement, while the extension asks them to construct an equation with a specified solution set. Both directions connect algebraic factors and zeros rather than only memorize a sign rule.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions; equation list on PDFp2; TE Student task, anticipated reasoning and Activity Synthesis


Take turns with your partner to match an equation with a graph or a description of a graph.
1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions #1–2

Teacher supports

Teacher guidance responds to factor-sign errors by asking students to substitute their proposed zero. For similar graphs, students are prompted to check another feature rather than accept the first apparent match.

TE 5.2 and 5.3 Anticipated Misconceptions; SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions; equation list on PDFp2

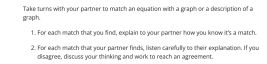

Anticipated Misconceptions | Students may mistakenly go from $x+a=0$ to $x=a$, from $2x+8=0$ to $x=8$, or $x=-8$. Encourage them to check their answer by substituting it for x in the original equation.

TE 5.2 Anticipated Misconceptions

Mathematical practices

MP7 is supported by recognizing how a product becomes zero and using that structure to locate intercepts. A zero is not generally the constant written in its linear factor with the same sign.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions; equation list on PDFp2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

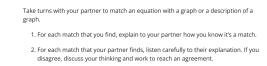

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1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions #1–2

Formative assessment

The teacher listens to matching explanations and probes whether students checked enough points or intercept information. Substitution can show whether a wrong match came from a sign error or insufficient graphical evidence.

SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions; equation list on PDFp2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


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SE 2.5 task · PDF p. 3, Activity5.3 partner explanation directions #1–2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

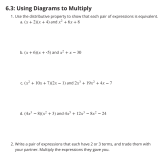
2.6 Different Forms

Explore lesson 2.6

Standards connection

Students verify equivalent expression pairs with distributive multiplication, then author expressions for a partner to multiply. This connects procedural execution with generated practice. HSA.APR.A.1/HSA.SSE.A.2 contribution: equivalent forms and products; supplied operation is weaker evidence than independently choosing a rewrite.

SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

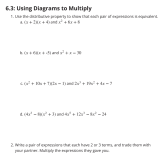


SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

Conceptual understanding

The same polynomial is examined in factored and standard forms to reveal different features. The box context and vertically scaled graphs connect those forms with intercepts, degree and output values.

SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

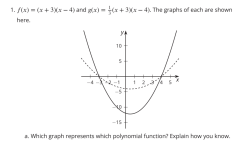


SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

Procedural skill and practice

Practice #1 matches two equations with their graphs by using their output scale and shared intercept structure. An explanation is needed because the graphs can share zeros without being the same function.

SE 2.6 practice · PDF p. 1, Practice #1

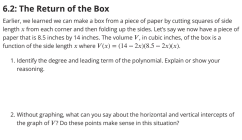


SE 2.6 practice · PDF p. 1, #1

Contextual application

Returning to the box makes equivalent expressions descriptions of the same volume function. Students should connect each factor to a dimension and interpret expanded form without changing the physical domain.

SE 2.6 task · PDF p. 1, Activity 6.2

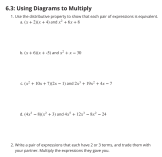


SE 2.6 task · PDF p. 1, Activity 6.2

Coherence: horizontal alignment

Return of the Box deliberately revisits the earlier volume model. Equivalent forms preserve that same volume function while exposing different features, so agreement is about all allowed inputs rather than a single evaluated cut size.

SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2; 2.1 task · PDF p. 2, Activities 1.2–1.3

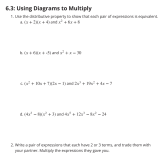


SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

Depth of reasoning

Students explain which information is easy to read from each form and predict what multiplying by a constant changes. Common zeros alone do not imply identical heights or vertical intercepts.

SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2; TE Student task, anticipated reasoning and Activity Synthesis

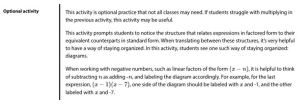


SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

Teacher supports

The teacher marks diagram-based multiplication practice optional when learners need additional support. Completed examples are reduced and alternative multiplication methods are invited, keeping equivalent-form reasoning as the goal.

TE 6.3 Launch and Activity Synthesis; Optional activity; SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

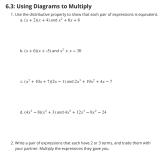


TE 6.3 Optional activity

Mathematical practices

MP7 is supported when students use factored form for zeros and standard form for degree or constant term. The useful form depends on the mathematical feature they need.

SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

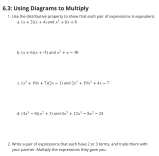


SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

Formative assessment

The teacher compares predictions about the scaled graphs with actual outputs and expanded expressions. Disagreement can expose confusion between changing a coefficient and changing the locations of zeros.

SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


6.3: Using Diagrams to Multiply

SE 2.6 task · PDF p. 2, Activity6.3 #1a–d,#2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

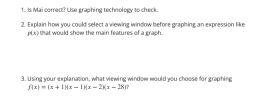
2.7 Using Factors and Zeros

Explore lesson 2.7

Standards connection

Students refute a polynomial-degree claim based on a misleading graph window, justify a better window and choose one for a new polynomial. HSF.IF.C.7c/HSA.APR.B.3: factors and zeros inform graph features and suitable viewing scales. Tool-setting decisions are visible; tool selection itself is prescribed.

SE 2.7 task · PDF p. 2, Activity7.2 #1–3; claim and graph on PDFp1


1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

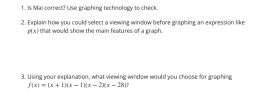
3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

SE 2.7 task · PDF p. 2, Activity 7.2

Conceptual understanding

Students use known factors and zeros to judge whether a graphing window shows the main features. A cubic may appear parabolic in a restricted view, so local appearance does not determine the function's degree.

SE 2.7 task · PDF p. 2, Activity7.2 #1–3; claim and graph on PDFp1


1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

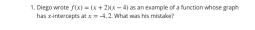
3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

SE 2.7 task · PDF p. 2, Activity 7.2

Procedural skill and practice

Practice #1 diagnoses an incorrect equation proposed for specified intercepts. Students must check the factor signs against the claimed zeros rather than accept a plausible-looking product.

SE 2.7 practice · PDF p. 1, Practice #1

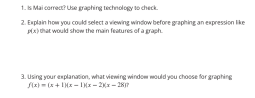

1. Diego wrote $f(x) = (x+2)(x-4)$ as an example of a function whose graph has x-intercepts at $x = -4, 2$. What was his mistake?

SE 2.7 practice · PDF p. 1, #1

Coherence: horizontal alignment

Lesson 5 reads zeros from factors; this lesson reverses the direction by constructing functions from desired zeros. The reverse task exposes that shared zeros do not uniquely determine the whole polynomial.

SE 2.7 task · PDF p. 2, Activity7.2 #1–3; claim and graph on PDFp1; 2.5 task · PDF p. 2, Activity 5.3


1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

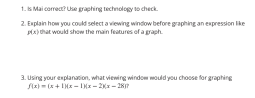
3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

SE 2.7 task · PDF p. 2, Activity 7.2

Depth of reasoning

Students refute Mai's graph-based claim, justify a suitable window and then construct possible equations from intercepts. Different valid equations can share those intercepts, so the answer need not be unique.

SE 2.7 task · PDF p. 2, Activity7.2 #1–3; claim and graph on PDFp1; TE Student task, anticipated reasoning and Activity Synthesis


1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

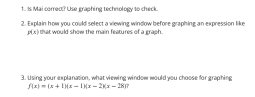
3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

SE 2.7 task · PDF p. 2, Activity 7.2

Teacher supports

The teacher asks which factor structure informed students' window choices and checks early progress on their equations. Sign mistakes are investigated by evaluating or graphing the proposed function.

TE 7.2 Activity Synthesis; 7.3 Anticipated Misconceptions; SE 2.7 task · PDF p. 2, Activity7.2 #1–3; claim and graph on PDFp1


1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

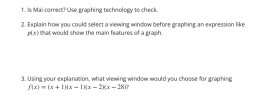
3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

SE 2.7 task · PDF p. 2, Activity 7.2

Mathematical practices

MP7 is supported by using factor structure to predict what the graph must show. Choosing a graph window is not counted here as independently selecting a tool for MP5.

SE 2.7 task · PDF p. 2, Activity7.2 #1–3; claim and graph on PDFp1; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

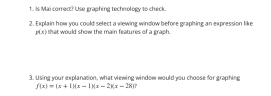
3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

SE 2.7 task · PDF p. 2, Activity 7.2

Formative assessment

The teacher compares the selected horizontal range with the zeros implied by the factors. This checks whether a window omitted an intercept and why the displayed graph led to a mistaken conclusion.

SE 2.7 task · PDF p. 2, Activity7.2 #1–3; claim and graph on PDFp1; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


1. Is Mai correct? Use graphing technology to check.
2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.
3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

SE 2.7 task · PDF p. 2, Activity 7.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

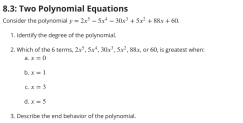
2.8 End Behavior (Part 1)

Explore lesson 2.8

Standards connection

Students identify degree, compare term values at specified inputs and explain polynomial end behavior. Eventually dominant leading behavior must be distinguished from values near zero. HSF.IF.C.7c: relate degree/leading term to ends, with numerical comparisons feeding graphical reasoning.

SE 2.8 task · PDF p. 3, Activity8.3 #1–3; Lesson8 Summary

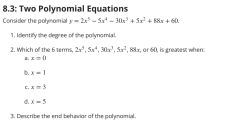

8.3: Two Polynomial Equations
Consider the polynomial $y = 2x^5 - 5x^4 - 30x^3 + 5x^2 + 88x + 60$.
1. Identify the degree of the polynomial.
2. Which of the 6 terms, $2x^5$, $5x^4$, $30x^3$, $5x^2$, $88x$, or 60, is greatest when:
a. $x = 0$
b. $x = 1$
c. $x = 3$
d. $x = 5$
3. Describe the end behavior of the polynomial.

SE 2.8 task · PDF p. 3, Activity 8.3

Conceptual understanding

Numerical tables and term comparisons develop the idea that the leading term determines eventual polynomial behavior. The dominant term far from zero need not be the largest contribution at every smaller input.

SE 2.8 task · PDF p. 3, Activity8.3 #1–3; Lesson8 Summary

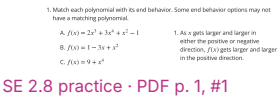

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a. $x = 0$
b. $x = 1$
c. $x = 3$
d. $x = 5$
3. Describe the end behavior of the polynomial.

SE 2.8 task · PDF p. 3, Activity 8.3

Procedural skill and practice

Practice #1 matches polynomial expressions with end-behavior descriptions. Students coordinate degree parity and leading sign rather than calculate a long list of outputs.

SE 2.8 practice · PDF p. 1, Practice #1

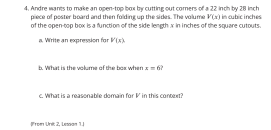

1. Match each polynomial with its end behavior. Some end behavior options may not have a matching polynomial.
A. $f(x) = 2x^3 + 3x^4 + x^2 - 1$
B. $f(x) = 1 - 3x + x^2$
C. $f(x) = 9 + x^4$
1. As x gets larger and larger in either the positive or negative direction, $f(x)$ gets larger and larger in the positive direction.

SE 2.8 practice · PDF p. 1, #1

Contextual application

The box application asks students to use a polynomial model while retaining its construction constraints. End behavior of the unrestricted polynomial does not by itself describe physically possible boxes.

SE 2.8 practice · PDF p. 2, Practice #4

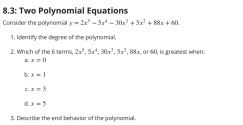

4. Andre wants to make an open-top box by cutting out corners of a 22 inch by 28 inch piece of poster board and then folding up the sides. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts.
a. Write an expression for $V(x)$.
b. What is the volume of the box when $x = 6$?
c. What is a reasonable domain for V in this context?
(From Unit 2, Lesson 1.)

SE 2.8 practice · PDF p. 2, Practice #4

Coherence: horizontal alignment

Constructing a polynomial with specified intercepts leaves its distant behavior unresolved. Comparing ends adds a global graph feature to the earlier local zero information.

SE 2.8 task · PDF p. 3, Activity8.3 #1–3; Lesson8 Summary; 2.7 task · PDF p. 2, Activity 7.3

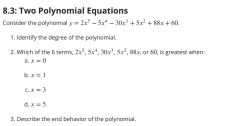

8.3: Two Polynomial Equations
Consider the polynomial $y = 2x^5 - 5x^4 - 30x^3 + 5x^2 + 88x + 60$.
1. Identify the degree of the polynomial.
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a. $x = 0$
b. $x = 1$
c. $x = 3$
d. $x = 5$
3. Describe the end behavior of the polynomial.

SE 2.8 task · PDF p. 3, Activity 8.3

Depth of reasoning

Activity 8.3 asks which term is greatest at several inputs before explaining end behavior. A convincing conclusion must account for large inputs in both directions, not infer a global pattern from a small graph window.

SE 2.8 task · PDF p. 3, Activity8.3 #1–3; Lesson8 Summary; TE Student task, anticipated reasoning and Activity Synthesis

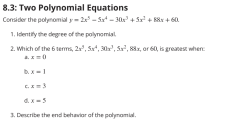

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a. $x = 0$
b. $x = 1$
c. $x = 3$
d. $x = 5$
3. Describe the end behavior of the polynomial.

SE 2.8 task · PDF p. 3, Activity 8.3

Teacher supports

Teacher guidance addresses the difference between raising a negative input to a power and negating a power. The synthesis also uses charts and revised explanations to connect degree with the two ends of a graph.

TE 8.2 Anticipated Misconceptions; 8.3 Activity Synthesis; SE 2.8 task · PDF p. 3, Activity8.3 #1–3; Lesson8 Summary

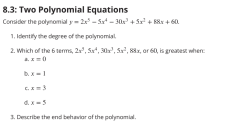

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a. $x = 0$
b. $x = 1$
c. $x = 3$
d. $x = 5$
3. Describe the end behavior of the polynomial.

SE 2.8 task · PDF p. 3, Activity 8.3

Mathematical practices

MP8 is supported by examining repeated evaluations and generalizing which term governs eventual behavior. Students must check that the generalization holds beyond the initially sampled inputs.

SE 2.8 task · PDF p. 3, Activity8.3 #1–3; Lesson8 Summary; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


8.3: Two Polynomial Equations

Consider the polynomial $y = 2x^5 - 5x^4 - 30x^3 + 5x^2 + 88x + 60$.

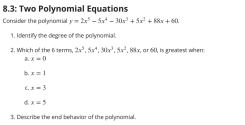
1. Identify the degree of the polynomial.
2. Which of the 6 terms, $2x^5$, $5x^4$, $30x^3$, $5x^2$, $88x$, or 60, is greatest when:
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   b. $x = 1$
   c. $x = 3$
   d. $x = 5$
3. Describe the end behavior of the polynomial.

SE 2.8 task · PDF p. 3, Activity 8.3

Formative assessment

The teacher reviews how learners entered negative values and compares their descriptions of even- and odd-degree behavior. This can distinguish a calculator-parentheses error from a mistaken generalization about polynomial ends.

SE 2.8 task · PDF p. 3, Activity8.3 #1–3; Lesson8 Summary; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


8.3: Two Polynomial Equations

Consider the polynomial $y = 2x^5 - 5x^4 - 30x^3 + 5x^2 + 88x + 60$.

1. Identify the degree of the polynomial.
2. Which of the 6 terms, $2x^5$, $5x^4$, $30x^3$, $5x^2$, $88x$, or 60, is greatest when:
   a. $x = 0$
   b. $x = 1$
   c. $x = 3$
   d. $x = 5$
3. Describe the end behavior of the polynomial.

SE 2.8 task · PDF p. 3, Activity 8.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

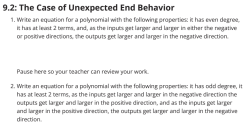
2.9 End Behavior (Part 2)

Explore lesson 2.9

Standards connection

Students construct even- and odd-degree polynomials satisfying stated end behaviors and explain their reasoning. Degree parity and leading sign both constrain valid answers. HSF.IF.C.7c: author functions from end-behavior conditions, rather than merely match an existing graph.

SE 2.9 task · PDF p. 1, Activity9.2 #1–2


9.2: The Case of Unexpected End Behavior

1. Write an equation for a polynomial with the following properties: it has even degree, it has at least 2 terms, and, as the inputs get larger and larger in either the negative or positive directions, the outputs get larger and larger in the negative direction.

Pause here so your teacher can review your work.

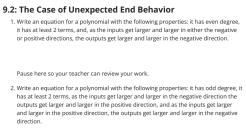
2. Write an equation for a polynomial with the following properties: it has odd degree, it has at least 2 terms, as the inputs get larger and larger in the negative direction the outputs get larger and larger in the positive direction, and as the inputs get larger and larger in the positive direction, the outputs get larger and larger in the negative direction.

SE 2.9 task · PDF p. 1, Activity 9.2

Conceptual understanding

Students extend degree-parity reasoning to polynomials with negative leading coefficients. Constructing examples makes clear that the sign and degree together control the two ends.

SE 2.9 task · PDF p. 1, Activity9.2 #1–2


9.2: The Case of Unexpected End Behavior

1. Write an equation for a polynomial with the following properties: it has even degree, it has at least 2 terms, and, as the inputs get larger and larger in either the negative or positive directions, the outputs get larger and larger in the negative direction.

Pause here so your teacher can review your work.

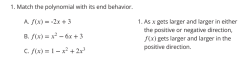
2. Write an equation for a polynomial with the following properties: it has odd degree, it has at least 2 terms, as the inputs get larger and larger in the negative direction the outputs get larger and larger in the positive direction, and as the inputs get larger and larger in the positive direction, the outputs get larger and larger in the negative direction.

SE 2.9 task · PDF p. 1, Activity 9.2

Procedural skill and practice

Practice #1 matches expressions with end-behavior descriptions for different leading signs and parities. The work practices coordinated classification rather than determining behavior from degree alone.

SE 2.9 practice · PDF p. 1, Practice #1


1. Match the polynomial with its end behavior.

A. $f(x) = -2x + 3$

B. $f(x) = x^2 - 6x + 3$

C. $f(x) = 1 - x^2 + 2x^3$

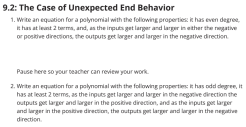
1. As x gets larger and larger in either the positive or negative direction, $f(x)$ gets larger and larger in the positive direction.

SE 2.9 practice · PDF p. 1, #1

Coherence: horizontal alignment

The preceding work organizes end behavior by degree parity. Negative leading coefficients now require students to revise the direction of the ends while retaining the parity distinction.

SE 2.9 task · PDF p. 1, Activity9.2 #1–2; 2.8 task · PDF p. 3, Activity 8.3 and Summary


9.2: The Case of Unexpected End Behavior

1. Write an equation for a polynomial with the following properties: it has even degree, it has at least 2 terms, and, as the inputs get larger and larger in either the negative or positive directions, the outputs get larger and larger in the negative direction.

Pause here so your teacher can review your work.

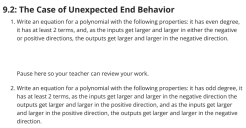
2. Write an equation for a polynomial with the following properties: it has odd degree, it has at least 2 terms, as the inputs get larger and larger in the negative direction the outputs get larger and larger in the positive direction, and as the inputs get larger and larger in the positive direction, the outputs get larger and larger in the negative direction.

SE 2.9 task · PDF p. 1, Activity 9.2

Depth of reasoning

The main task requires an equation satisfying several end-behavior conditions. Students must justify why their leading term produces those conditions and recognize that many equations can work.

SE 2.9 task · PDF p. 1, Activity9.2 #1–2; TE Student task, anticipated reasoning and Activity Synthesis


9.2: The Case of Unexpected End Behavior

1. Write an equation for a polynomial with the following properties: it has even degree, it has at least 2 terms, and, as the inputs get larger and larger in either the negative or positive directions, the outputs get larger and larger in the negative direction.

Pause here so your teacher can review your work.

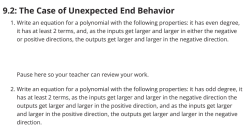
2. Write an equation for a polynomial with the following properties: it has odd degree, it has at least 2 terms, as the inputs get larger and larger in the negative direction the outputs get larger and larger in the positive direction, and as the inputs get larger and larger in the positive direction, the outputs get larger and larger in the negative direction.

SE 2.9 task · PDF p. 1, Activity 9.2

Teacher supports

The teacher begins some comparisons without graphing technology so students reason from expression structure. Learners who cannot start may sketch the required shape before choosing an equation.

TE 9.1 and 9.2 Launch; SE 2.9 task · PDF p. 1, Activity9.2 #1–2


9.2: The Case of Unexpected End Behavior

1. Write an equation for a polynomial with the following properties: it has even degree, it has at least 2 terms, and, as the inputs get larger and larger in either the negative or positive directions, the outputs get larger and larger in the negative direction.

Pause here so your teacher can review your work.

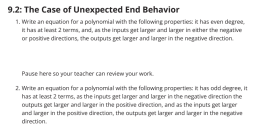
2. Write an equation for a polynomial with the following properties: it has odd degree, it has at least 2 terms, as the inputs get larger and larger in the negative direction the outputs get larger and larger in the positive direction, and as the inputs get larger and larger in the positive direction, the outputs get larger and larger in the negative direction.

SE 2.9 task · PDF p. 1, Activity 9.2

Mathematical practices

MP7 is supported by using the leading coefficient and degree to generate a function with desired behavior. Checking a few points alone does not justify the eventual directions.

SE 2.9 task · PDF p. 1, Activity9.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


9.2: The Case of Unexpected End Behavior

1. Write an equation for a polynomial with the following properties: it has even degree, it has at least 2 terms, and, as the inputs get larger and larger in either the negative or positive directions, the outputs get larger and larger in the negative direction.

Pause here so your teacher can review your work.

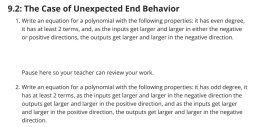
2. Write an equation for a polynomial with the following properties: it has odd degree, it has at least 2 terms, as the inputs get larger and larger in the negative direction the outputs get larger and larger in the positive direction, and as the inputs get larger and larger in the positive direction, the outputs get larger and larger in the negative direction.

SE 2.9 task · PDF p. 1, Activity 9.2

Formative assessment

The teacher compares the different equations groups created and asks what their leading terms have in common. An incorrect direction at one end reveals whether sign or parity was used inconsistently.

SE 2.9 task · PDF p. 1, Activity9.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


9.2: The Case of Unexpected End Behavior

1. Write an equation for a polynomial with the following properties: it has even degree, it has at least 2 terms, and, as the inputs get larger and larger in either the negative or positive directions, the outputs get larger and larger in the negative direction.

Pause here so your teacher can review your work.

2. Write an equation for a polynomial with the following properties: it has odd degree, it has at least 2 terms, as the inputs get larger and larger in the negative direction the outputs get larger and larger in the positive direction, and as the inputs get larger and larger in the positive direction, the outputs get larger and larger in the negative direction.

SE 2.9 task · PDF p. 1, Activity 9.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

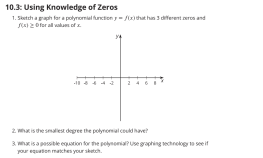
2.10 Multiplicity

Explore lesson 2.10

Standards connection

Students construct a nonnegative polynomial with three distinct zeros, determine minimum degree and test a possible equation. Extension asks why some crossing/degree requirements are impossible. HSA.APR.B.3/HSF.IF.C.7c: combine zeros, multiplicity, parity and sign; zero count alone does not determine degree.

SE 2.10 task · PDF p. 4, Activity10.3 #1–3; Are you ready for more?


10.3: Using Knowledge of Zeros

1. Sketch a graph for a polynomial function $y = f(x)$ that has 3 different zeros and $f(x) \geq 0$ for all values of x.

2. What is the smallest degree the polynomial could have?

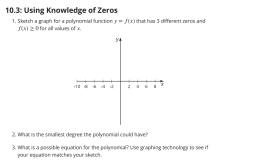
3. What is a possible equation for the polynomial? Use graphing technology to see if your equation matches your sketch.

SE 2.10 task · PDF p. 4, Activity 10.3

Conceptual understanding

Multiplicity adds local graph behavior near a zero to the previously learned zeros and end behavior. Students combine these features when constructing a rough sketch instead of treating all intercepts as simple crossings.

SE 2.10 task · PDF p. 4, Activity10.3 #1–3; Are you ready for more?


10.3: Using Knowledge of Zeros

1. Sketch a graph for a polynomial function $y = f(x)$ that has 3 different zeros and $f(x) \geq 0$ for all values of x.

2. What is the smallest degree the polynomial could have?

3. What is a possible equation for the polynomial? Use graphing technology to see if your equation matches your sketch.

SE 2.10 task · PDF p. 4, Activity 10.3

Procedural skill and practice

Practice #1 sketches a polynomial from its factored form. The factors and degree constrain intercepts and global direction; the sketch need not calculate every turning point exactly.

SE 2.10 practice · PDF p. 1, Practice #1

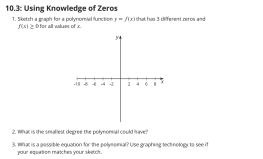

1. Draw a rough sketch of the graph of $g(x) = (x-3)(x+1)(7x-2)$.

SE 2.10 practice · PDF p. 1, #1

Coherence: horizontal alignment

End behavior describes what happens far from the zeros; multiplicity describes the graph near a zero. The lesson combines these complementary features when interpreting factored polynomials.

SE 2.10 task · PDF p. 4, Activity10.3 #1–3; Are you ready for more?; 2.9 task · PDF p. 1, Activity 9.2


10.3: Using Knowledge of Zeros

1. Sketch a graph for a polynomial function $y = f(x)$ that has 3 different zeros and $f(x) \geq 0$ for all values of x.

2. What is the smallest degree the polynomial could have?

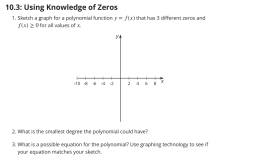
3. What is a possible equation for the polynomial? Use graphing technology to see if your equation matches your sketch.

SE 2.10 task · PDF p. 4, Activity 10.3

Depth of reasoning

The optional task asks for a nonnegative polynomial with three distinct zeros and its smallest possible degree. Students must explain why touching the axis differs from crossing it; the extension tests an impossible degree-and-crossing combination.

SE 2.10 task · PDF p. 4, Activity10.3 #1–3; Are you ready for more?; TE Student task, anticipated reasoning and Activity Synthesis


10.3: Using Knowledge of Zeros

1. Sketch a graph for a polynomial function $y = f(x)$ that has 3 different zeros and $f(x) \geq 0$ for all values of x.

2. What is the smallest degree the polynomial could have?

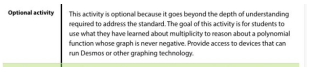
3. What is a possible equation for the polynomial? Use graphing technology to see if your equation matches your sketch.

SE 2.10 task · PDF p. 4, Activity 10.3

Teacher supports

The teacher explicitly labels Activity 10.3 as beyond the depth required by the standard. For the main sketches, selected practice examples and sign testing support learners without requiring unnecessary exact-output arithmetic.

TE 10.3 Optional activity; SE 2.10 task · PDF p. 4, Activity10.3 #1–3; Are you ready for more?

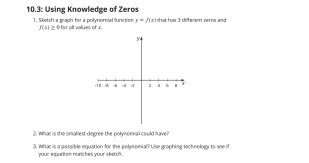


TE 10.3 Optional activity

Mathematical practices

MP7 is supported by coordinating multiplicity, zeros, degree and sign in a graph. The reasoning uses the structure of repeated factors, not just the number of different intercepts.

SE 2.10 task · PDF p. 4, Activity10.3 #1–3; Are you ready for more?; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

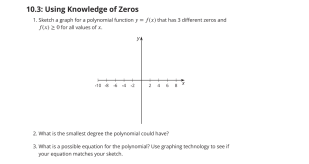


SE 2.10 task · PDF p. 4, Activity 10.3

Formative assessment

The teacher asks which earlier sketching strategies were useful and checks sketches with technology. Comparing signs on either side of a repeated factor can reveal why an intercept should touch or cross.

SE 2.10 task · PDF p. 4, Activity10.3 #1–3; Are you ready for more?; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.10 task · PDF p. 4, Activity 10.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

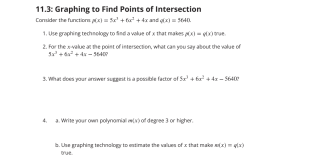
2.11 Finding Intersections

Explore lesson 2.11

Standards connection

Students estimate equal-output inputs graphically, connect their difference to zero and a possible factor, then author a higher-degree polynomial for a new comparison. HSA.REI.D.11: equation solutions correspond to intersection input values. Other function families in full standard are not established by polynomial-only task.

SE 2.11 task · PDF p. 3, Activity11.3 #1–4

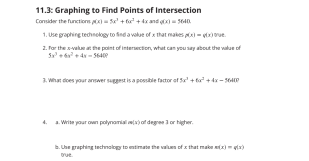


SE 2.11 task · PDF p. 3, Activity 11.3

Conceptual understanding

An intersection represents equal outputs for the same input, so its input coordinate solves the corresponding equation. Students connect this graph interpretation with a zero of the difference between two functions.

SE 2.11 task · PDF p. 3, Activity11.3 #1–4



SE 2.11 task · PDF p. 3, Activity 11.3

Procedural skill and practice

Practice #1 finds intersections of two polynomial graphs from their equations. Solving for the input must be followed by finding the common output to report a point.

SE 2.11 practice · PDF p. 1, Practice #1

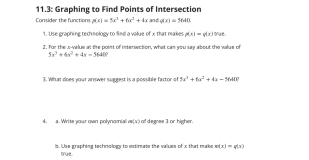


SE 2.11 practice · PDF p. 1, #1

Coherence: horizontal alignment

A polynomial previously used for place-value evaluation returns in an intersection problem. Equating two expressions and examining their difference connects numerical coincidences with factors and shared graph points.

SE 2.11 task · PDF p. 3, Activity11.3 #1–4; 2.2 task · PDF p. 1, Activity 2.2

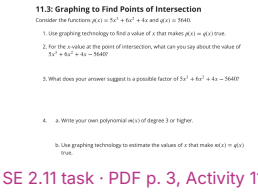


SE 2.11 task · PDF p. 3, Activity 11.3

Depth of reasoning

The task connects an approximate intersection with a possible factor of a polynomial difference. Dividing away a shared variable factor can lose another solution, so algebraic transformations need justification.

SE 2.11 task · PDF p. 3, Activity11.3 #1–4; TE Student task, anticipated reasoning and Activity Synthesis

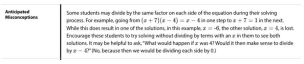

11.3: Graphing to Find Points of Intersection

SE 2.11 task · PDF p. 3, Activity 11.3

Teacher supports

The teacher explicitly addresses lost solutions when students divide by a factor that could be zero. Students are asked to solve without that division and consider why dividing by zero would be invalid.

TE 11.2 Anticipated Misconceptions; SE 2.11 task · PDF p. 3, Activity11.3 #1–4

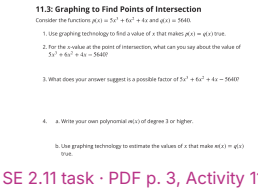

Anticipated Misconceptions

TE 11.2 Anticipated Misconceptions

Mathematical practices

MP7 is supported by recognizing that equal-output inputs are zeros of a difference. This connection can guide both algebraic solving and approximate graphical checks.

SE 2.11 task · PDF p. 3, Activity11.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

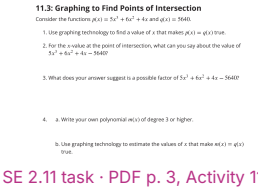

11.3: Graphing to Find Points of Intersection

SE 2.11 task · PDF p. 3, Activity 11.3

Formative assessment

The teacher selects different solution methods and compares the resulting intersection sets. A missing point can then be traced to cancelling a possibly zero factor rather than dismissed as a graphing mistake.

SE 2.11 task · PDF p. 3, Activity11.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


11.3: Graphing to Find Points of Intersection

SE 2.11 task · PDF p. 3, Activity 11.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

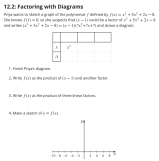
2.12 Polynomial Division (Part 1)

Explore lesson 2.12

Standards connection

Students finish a multiplication diagram, recover missing factors, fully factor and sketch the polynomial graph. The diagram is a representation of division, not an unrelated illustration. HSA.APR.B.2/B.3: check a suspected factor and use resulting zeros in graph construction; full remainder generalization comes later.

SE 2.12 task · PDF p. 2, Activity12.2 #1–4

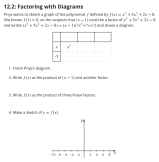

12.2: Factoring with Diagrams

SE 2.12 task · PDF p. 2, Activity 12.2

Conceptual understanding

The multiplication diagram is used in reverse to recover a missing polynomial factor. Students connect the completed product, the factorization and a graph determined by the resulting zeros.

SE 2.12 task · PDF p. 2, Activity12.2 #1–4

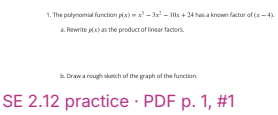

12.2: Factoring with Diagrams

SE 2.12 task · PDF p. 2, Activity 12.2

Procedural skill and practice

Practice #1 uses a known factor to rewrite a polynomial as linear factors and draw a rough graph. The division result must be factored further before all zeros are visible.

SE 2.12 practice · PDF p. 1, Practice #1

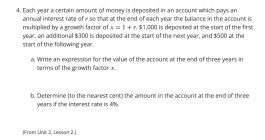

1. The polynomial function $p(x) = x^3 - 3x^2 - 10x + 24$ has a known factor of $(x - 4)$.

SE 2.12 practice · PDF p. 1, #1

Contextual application

The investment practice connects an algebraic expression to account value. A proposed factor or solution needs interpretation in the stated timing and rate setting rather than being accepted solely because the algebra works.

SE 2.12 practice · PDF p. 2, Practice #4


4. Each year a certain amount of money is deposited in an account which pays an annual interest rate of r so that at the end of each year the balance in the account is multiplied by a growth factor of $x = 1 + r$. $1,000 is deposited at the start of the first year, an additional $300 is deposited at the start of the next year, and $500 at the start of the following year.

a. Write an expression for the value of the account at the end of three years in terms of the growth factor x.

b. Determine (to the nearest cent) the amount in the account at the end of three years if the interest rate is 4%.

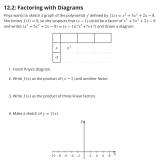
(From Unit 2, Lesson 2.)

SE 2.12 practice · PDF p. 2, Practice #4

Coherence: horizontal alignment

Earlier tasks read zeros from already factored expressions. Here students use suspected zeros to recover factors from standard form, reversing the previous representation change.

SE 2.12 task · PDF p. 2, Activity12.2 #1–4; 2.5 task · PDF p. 3, Lesson Summary

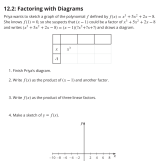

12.2: Factoring with Diagrams

SE 2.12 task · PDF p. 2, Activity 12.2

Depth of reasoning

Students explain how missing diagram terms are forced by the product. A suspected factor is checked by completing the multiplication relationship, not accepted solely because one evaluation appears to be zero.

SE 2.12 task · PDF p. 2, Activity12.2 #1–4; TE Student task, anticipated reasoning and Activity Synthesis

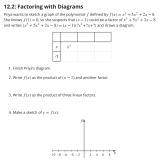

12.2: Factoring with Diagrams

SE 2.12 task · PDF p. 2, Activity 12.2

Teacher supports

The teacher connects diagram cells with products and clarifies why the last cell need not contain zero. When further division is needed, learners are reminded to use the quotient rather than restart with the original polynomial.

TE 12.2 and 12.3 Anticipated Misconceptions; SE 2.12 task · PDF p. 2, Activity12.2 #1–4

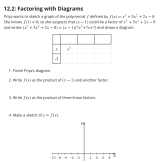

12.2: Factoring with Diagrams

SE 2.12 task · PDF p. 2, Activity 12.2

Mathematical practices

MP7 is supported by reconstructing a factor from the structure of a known product. The diagram organizes reasoning about multiplication and division rather than serving as a decorative model.

SE 2.12 task · PDF p. 2, Activity12.2 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

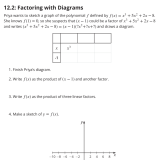

12.2: Factoring with Diagrams

SE 2.12 task · PDF p. 2, Activity 12.2

Formative assessment

Teachers compare partner diagrams and ask learners to account for particular signs and terms. This can separate a diagram-placement error from an incorrect factor or multiplication.

SE 2.12 task · PDF p. 2, Activity12.2 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


12.2: Factoring with Diagrams

SE 2.12 task · PDF p. 2, Activity 12.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

2.13 Polynomial Division (Part 2)

Explore lesson 2.13

Standards connection

Students finish polynomial long division and translate a completed multiplication diagram into long division, before independently factoring later examples. HSA.APR.D.6/B.2 contribution: connect division representations and reduce scaffolding; rational-expression rewriting still requires remainder/degree constraints.

SE 2.13 task · PDF p. 2, Activity13.2 #2–3; continuation PDFp3

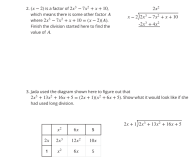


SE 2.13 task · PDF p. 2, Activity 13.2

Conceptual understanding

Students connect polynomial long division with the diagram method already used for the same products. Aligning terms by variable degree makes the subtraction structure visible.

SE 2.13 task · PDF p. 2, Activity13.2 #2–3; continuation PDFp3

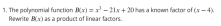


SE 2.13 task · PDF p. 2, Activity 13.2

Procedural skill and practice

Practice #1 rewrites a polynomial with a known factor as a product of linear factors. Long division identifies the quotient, which may still require additional factoring.

SE 2.13 practice · PDF p. 1, Practice #1



SE 2.13 practice · PDF p. 1, #1

Coherence: vertical alignment

The summary explicitly connects polynomial division with the integer long division learned in earlier grades. Students use place-value reasoning with powers of the variable; the connection is more specific than merely sharing the word division.

SE 2.13 task · PDF p. 5, Lesson Summary: earlier-grade integer long division and polynomial division

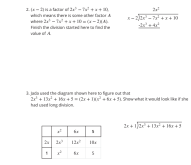


SE 2.13 task · PDF p. 5, Lesson Summary: earlier-grade integer long division and polynomial division

Coherence: horizontal alignment

Diagrams used to identify a missing polynomial factor precede the long-division algorithm. Comparing the two records gives meaning to each quotient term and subtraction.

SE 2.13 task · PDF p. 2, Activity13.2 #2–3; continuation PDFp3; 2.12 task · PDF p. 3, Activity 12.3

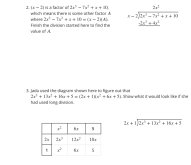


SE 2.13 task · PDF p. 2, Activity 13.2

Depth of reasoning

Translating a completed division into a diagram and a diagram into long division requires explaining the same product in two ways. The optional missing-number problems ask students to use equivalence rather than repeat an identical algorithm.

SE 2.13 task · PDF p. 2, Activity13.2 #2–3; continuation PDFp3; TE Student task, anticipated reasoning and Activity Synthesis

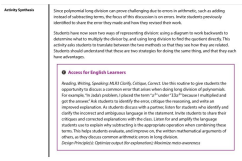


SE 2.13 task · PDF p. 2, Activity 13.2

Teacher supports

The synthesis focuses on arithmetic errors such as adding instead of subtracting a row and asks students to explain how they revised their work. The teacher then connects the direct quotient method with working backward through a diagram.

TE 13.2 Activity Synthesis; SE 2.13 task · PDF p. 2, Activity13.2 #2–3; continuation PDFp3

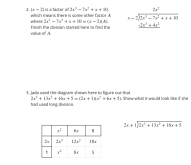


TE 13.2 Activity Synthesis

Mathematical practices

MP6 is supported by accurate degree alignment, signs and communication of subtraction steps. A correct quotient is more meaningful when learners can explain how each row contributes to it.

SE 2.13 task · PDF p. 2, Activity13.2 #2–3; continuation PDFp3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

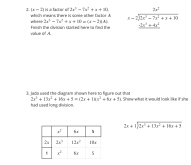


SE 2.13 task · PDF p. 2, Activity 13.2

Formative assessment

The teacher invites students who made an error to describe it and the revision that corrected it. Comparing this work with the diagram gives a way to check the procedure beyond inspecting a final quotient.

SE 2.13 task · PDF p. 2, Activity13.2 #2–3; continuation PDFp3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.13 task · PDF p. 2, Activity 13.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

2.14 What Do You Know About Polynomials?

Explore lesson 2.14

Standards connection

Students author a polynomial, graph it, change to standard form, exchange partial factor information and reconstruct a partner polynomial and graph, then reconcile differences. HSA.APR.B.3/HSF.IF.C.7c: bidirectional links among forms, zeros and graph; independent generation rather than only recognition.

SE 2.14 task · PDF p. 3, Activity14.3 #1–4



SE 2.14 task · PDF p. 3, Activity 14.3

Conceptual understanding

This synthesis brings together polynomial factors, standard form and graph features. Students create their own example and reconstruct a partner's example from partial information, reversing the direction of earlier recognition tasks.

SE 2.14 task · PDF p. 3, Activity14.3 #1–4

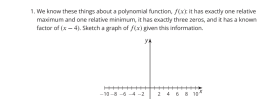


SE 2.14 task · PDF p. 3, Activity 14.3

Procedural skill and practice

Practice #1 constructs a graph from stated extrema, zeros and a known factor. The conditions constrain a possible sketch but need not determine a unique polynomial.

SE 2.14 practice · PDF p. 1, Practice #1



SE 2.14 practice · PDF p. 1, #1

Coherence: horizontal alignment

Known-factor division from Lesson 13 becomes a reconstruction problem with less information. A candidate polynomial must satisfy both the algebraic conditions and the stated graph features.

SE 2.14 task · PDF p. 3, Activity14.3 #1–4; 2.13 task · PDF p. 3, Activity 13.3



SE 2.14 task · PDF p. 3, Activity 14.3

Depth of reasoning

In Activity 14.3, students must reconcile their reconstructed factorization and graph with the partner's original. A disagreement requires checking mathematical constraints, not merely choosing the more attractive sketch.

SE 2.14 task · PDF p. 3, Activity14.3 #1–4; TE Student task, anticipated reasoning and Activity Synthesis



SE 2.14 task · PDF p. 3, Activity 14.3

Teacher supports

The info-gap launch assigns separate information roles and requires students to explain why they need a fact. The later polynomial-authoring activity is optional extra practice rather than a mandatory test of every learner.

TE 14.2 Launch; 14.3 Optional activity; SE 2.14 task · PDF p. 3, Activity14.3 #1–4



SE 2.14 task · PDF p. 3, Activity 14.3

Mathematical practices

MP3 is supported by comparing a reconstructed polynomial and graph with the original and discussing differences. The result must agree mathematically even if the algebraic forms look different.

SE 2.14 task · PDF p. 3, Activity14.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.14 task · PDF p. 3, Activity 14.3

Formative assessment

The teacher reviews work before providing another info-gap set, then asks students to explain their method to an absent classmate. These checkpoints reveal which information and factor relationships students actually used.

SE 2.14 task · PDF p. 3, Activity14.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.14 task · PDF p. 3, Activity 14.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

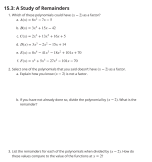
2.15 The Remainder Theorem

Explore lesson 2.15

Standards connection

Students identify possible factors, justify a nonfactor, divide to find its remainder and compare all remainders with function evaluation. Summary on PDFp4 supplies general converse/theorem proof. HSA.APR.B.2: zero/factor equivalence and nonzero evaluation/remainder relation, not only factor guessing.

SE 2.15 task · PDF p. 3, Activity15.3 #1a–f,#2a–b,#3

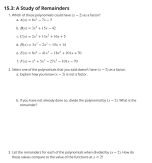


SE 2.15 task · PDF p. 3, Activity 15.3

Conceptual understanding

The Remainder Theorem connects function evaluation with polynomial division, including nonzero remainders. The lesson also establishes both directions of the relationship between a zero and a linear factor.

SE 2.15 task · PDF p. 3, Activity15.3 #1a–f,#2a–b,#3

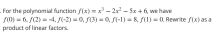


SE 2.15 task · PDF p. 3, Activity 15.3

Procedural skill and practice

Practice #1 uses given evaluations to identify zeros and rewrite a polynomial as linear factors. A zero evaluation supplies factor information that still needs to be used in the factorization.

SE 2.15 practice · PDF p. 1, Practice #1

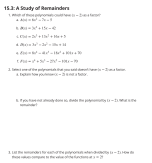


SE 2.15 practice · PDF p. 1, #1

Coherence: horizontal alignment

Previously a zero suggested a factor worth testing. The remainder and factor theorems explain why evaluation, remainder and divisibility are linked, making the earlier strategy general.

SE 2.15 task · PDF p. 3, Activity15.3 #1a–f,#2a–b,#3; 2.12 task · PDF p. 2, Activity 12.2

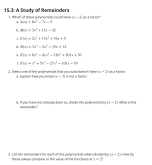


SE 2.15 task · PDF p. 3, Activity 15.3

Depth of reasoning

Students justify why a proposed expression is not a factor and compare division remainders with evaluations. The general statement is stronger than observing that one successful division happened to have no remainder.

SE 2.15 task · PDF p. 3, Activity15.3 #1a–f,#2a–b,#3; TE Student task, anticipated reasoning and Activity Synthesis

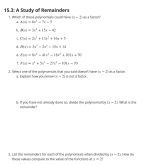


SE 2.15 task · PDF p. 3, Activity 15.3

Teacher supports

The teacher sequences different methods for finding an unknown coefficient and offers an example with more accessible values if needed. The later comparison keeps the focus on the remainder–evaluation relationship rather than only on division fluency.

TE 15.2 Launch and Activity Synthesis; 15.3; SE 2.15 task · PDF p. 3, Activity15.3 #1a–f,#2a–b,#3

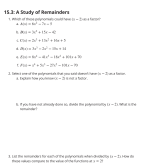


SE 2.15 task · PDF p. 3, Activity 15.3

Mathematical practices

MP8 is supported when repeated divisions and evaluations reveal a common relationship and students examine nonfactor cases. The theorem's general justification cannot be replaced by a list of confirming examples.

SE 2.15 task · PDF p. 3, Activity15.3 #1a–f,#2a–b,#3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

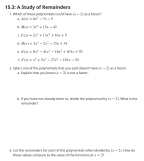


SE 2.15 task · PDF p. 3, Activity 15.3

Formative assessment

After initial work, the teacher compares groups' factor decisions and the remainders they found. Conflicting decisions can be checked by evaluation before students generalize the theorem.

SE 2.15 task · PDF p. 3, Activity15.3 #1a–f,#2a–b,#3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.15 task · PDF p. 3, Activity 15.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

2.16 Minimizing Surface Area

Explore lesson 2.16

Standards connection

Students explore how radius changes surface area with fixed cylinder volume, graph the relationship and write an area function. Positive dimensions constrain the physical domain. HSA.CED.A.4/HSF.IF.B.5: eliminate height through fixed volume and interpret rational relationship; provided table/tool scaffolds formulation.

SE 2.16 task · PDF p. 3, Activity16.3 #1–4



SE 2.16 task · PDF p. 3, Activity 16.3

Conceptual understanding

Fixing a cylinder's volume makes height depend on radius, which then determines surface area. Students combine formulas to express one varying quantity in terms of another and interpret the resulting rational function.

SE 2.16 task · PDF p. 3, Activity16.3 #1–4

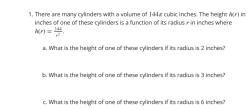


SE 2.16 task · PDF p. 3, Activity 16.3

Procedural skill and practice

Practice #1 evaluates cylinder height at several radii using a supplied fixed-volume relation. The procedure must preserve the squared-radius denominator and the units of height.

SE 2.16 practice · PDF p. 1, Practice #1



SE 2.16 practice · PDF p. 1, #1

Contextual application

Students express cylinder surface area while holding volume fixed. Substituting the constraint creates a rational function whose positive radius domain is essential when comparing designs.

SE 2.16 task · PDF p. 3, Activity 16.3



SE 2.16 task · PDF p. 3, Activity 16.3

Coherence: horizontal alignment

The earlier box problem maximizes volume under a construction constraint; the cylinder problem fixes volume and compares surface area. Solving the constraint introduces a rational expression with a meaningful positive domain.

SE 2.16 task · PDF p. 3, Activity16.3 #1–4; 2.1 task · PDF p. 2, Activity 1.3



SE 2.16 task · PDF p. 3, Activity 16.3

Depth of reasoning

The main task asks how radius affects area and requires an equation after numerical and graphical exploration. The can-design extension challenges the assumption that minimum material is the only manufacturing concern.

SE 2.16 task · PDF p. 3, Activity16.3 #1–4; TE Student task, anticipated reasoning and Activity Synthesis



SE 2.16 task · PDF p. 3, Activity 16.3

Teacher supports

Teacher guidance uses poured liquids or a physical can to connect symbols with volume and surface area. It also addresses exponent-grouping errors and helps students eliminate height using the volume relation.

TE 16.2 Launch and Anticipated Misconceptions; 16.3; SE 2.16 task · PDF p. 3, Activity16.3 #1–4



SE 2.16 task · PDF p. 3, Activity 16.3

Mathematical practices

MP2 is supported by coordinating radius, height, fixed volume and surface-area units. An algebraic value for a nonpositive physical dimension does not describe a usable can.

SE 2.16 task · PDF p. 3, Activity16.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.16 task · PDF p. 3, Activity 16.3

Formative assessment

The teacher compares computed heights and the shape of students' graphs before the surface-area model is assembled. This can expose a formula or exponent error before it distorts the optimization conclusion.

SE 2.16 task · PDF p. 3, Activity16.3 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.16 task · PDF p. 3, Activity 16.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

2.17 Graphs of Rational Functions (Part 1)

Explore lesson 2.17

Standards connection

Students create and graph travel time as a function of rate, interpret negative and zero inputs and reason about near-zero and long-run behavior. HSA.CED.A.2/HSF.IF.B.4/B.5: connect rational expression, graph and feasible physical quantities.

SE 2.17 task · PDF p. 2, Activity17.2 #1–6



SE 2.17 task · PDF p. 2, Activity 17.2

Conceptual understanding

Time and rate for a fixed trip form a rational relationship whose graph has meaningful limiting behavior. Students interpret zero and negative rates in context instead of treating every algebraic input as a feasible bicycle speed.

SE 2.17 task · PDF p. 2, Activity17.2 #1–6

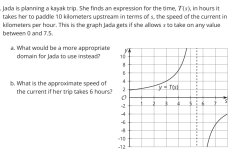


SE 2.17 task · PDF p. 2, Activity 17.2

Procedural skill and practice

Practice #1 uses a kayaking time graph to identify an appropriate current-speed domain and estimate a speed for a given travel time. Students must read the physical branch of the graph rather than any visible branch.

SE 2.17 practice · PDF p. 1, Practice #1



SE 2.17 practice · PDF p. 1, #1

Contextual application

The biking task relates rate and travel time. Students interpret why zero rate is excluded and why a mathematically drawn branch may be outside the situation.

SE 2.17 task · PDF p. 2, Activity 17.2



SE 2.17 task · PDF p. 2, Activity 17.2

Coherence: horizontal alignment

Rate and time calculations in the warm-up supply quantities for the biking graph. The graph must then reflect the division relationship and its excluded input, not simply join recorded values.

SE 2.17 task · PDF p. 2, Activity17.2 #1–6; p. 1, Warm-up 17.1



SE 2.17 task · PDF p. 2, Activity 17.2

Depth of reasoning

Students explain why travel time grows near zero rate and decreases as rate increases. The graph's separate branches need to be distinguished from the subset that answers the travel question.

SE 2.17 task · PDF p. 2, Activity17.2 #1–6; TE Student task, anticipated reasoning and Activity Synthesis



SE 2.17 task · PDF p. 2, Activity 17.2

Teacher supports

The teacher redirects distraction with negative graph inputs toward the contextual domain. For the rational-function card sort, color coding links asymptotes with denominator restrictions and different matching methods are compared.

TE 17.2 Anticipated Misconceptions; 17.3 Activity Synthesis; SE 2.17 task · PDF p. 2, Activity17.2 #1–6



SE 2.17 task · PDF p. 2, Activity 17.2

Mathematical practices

MP2 is supported by interpreting the symbolic and graphical behavior as travel time and speed. The meaning of the units and permitted rates guides which results are reasonable.

SE 2.17 task · PDF p. 2, Activity17.2 #1–6; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.17 task · PDF p. 2, Activity 17.2

Formative assessment

The teacher checks whether students can describe asymptotes and use them, along with intercepts, to justify card matches. A match based only on one point can be challenged with another defining feature.

SE 2.17 task · PDF p. 2, Activity17.2 #1–6; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.17 task · PDF p. 2, Activity 17.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

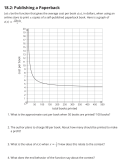
2.18 Graphs of Rational Functions (Part 2)

Explore lesson 2.18

Standards connection

Students interpret average book-printing cost, estimate profit threshold, question half-book input and explain end behavior in context. HSF.IF.B.4/B.5: graph features and quantitative domain; later rewriting links asymptote to expression form under HSA.APR.D.6.

SE 2.18 task · PDF p. 2, Activity18.2 #1–4

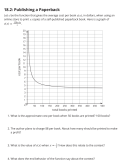


SE 2.18 task · PDF p. 2, Activity 18.2

Conceptual understanding

The average book-cost function introduces a nonzero horizontal asymptote. Rewriting and graph interpretation show why the long-run cost approaches a fixed amount rather than necessarily approaching zero.

SE 2.18 task · PDF p. 2, Activity18.2 #1–4

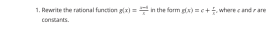


SE 2.18 task · PDF p. 2, Activity 18.2

Procedural skill and practice

Practice #1 rewrites a rational expression as a constant plus a remainder term. This makes the end behavior easier to read while retaining the original denominator restrictions.

SE 2.18 practice · PDF p. 1, Practice #1

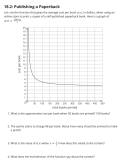


SE 2.18 practice · PDF p. 1, #1

Contextual application

Publishing cost per copy combines fixed and per-copy costs. The horizontal asymptote represents a limiting average cost, not an attainable zero total cost or a fixed number of copies.

SE 2.18 task · PDF p. 2, Activity 18.2

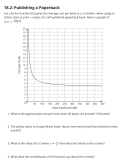


SE 2.18 task · PDF p. 2, Activity 18.2

Coherence: horizontal alignment

The preceding lesson introduces a vertical asymptote through an excluded input. Publishing cost now adds long-run behavior and a nonzero horizontal asymptote, requiring students to keep two different limiting questions separate.

SE 2.18 task · PDF p. 2, Activity18.2 #1–4; 2.17 task · PDF p. 4, Lesson Summary

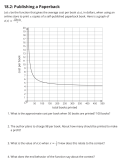


SE 2.18 task · PDF p. 2, Activity 18.2

Depth of reasoning

Students interpret the printing-cost graph, estimate a profit threshold and decide whether a half-book input makes sense. A graph alone can suggest a misleading limiting value, so numerical or algebraic checks matter.

SE 2.18 task · PDF p. 2, Activity18.2 #1–4; TE Student task, anticipated reasoning and Activity Synthesis

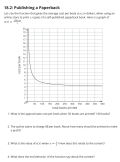


SE 2.18 task · PDF p. 2, Activity 18.2

Teacher supports

If students think the cost approaches zero from the displayed graph, the teacher suggests large inputs or a table. Color coding then connects an equivalent expression with the nonzero limiting constant.

TE 18.2 Anticipated Misconceptions and Activity Synthesis; SE 2.18 task · PDF p. 2, Activity18.2 #1–4

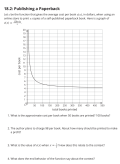


SE 2.18 task · PDF p. 2, Activity 18.2

Mathematical practices

MP2 is supported by relating average dollars per book, print quantity and the graph's end behavior. The algebraic function can accept inputs that do not represent a possible print run.

SE 2.18 task · PDF p. 2, Activity18.2 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

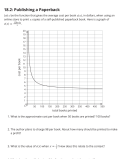


SE 2.18 task · PDF p. 2, Activity 18.2

Formative assessment

The teacher asks students to explain where the horizontal asymptote appears in the expressions and compare matching strategies. Their explanations distinguish recognizing a graph feature from understanding why it occurs.

SE 2.18 task · PDF p. 2, Activity18.2 #1–4; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.18 task · PDF p. 2, Activity 18.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

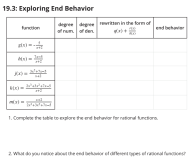
2.19 End Behavior of Rational Functions

Explore lesson 2.19

Standards connection

Students rewrite rational functions into quotient/remainder form and compare degree and end behavior; extension constructs a function approaching a nonhorizontal line. HSA.APR.D.6: proper remainder degree supports eventual approach to quotient; not all rational functions have horizontal asymptotes.

SE 2.19 task · PDF p. 2, Activity19.3 #1–2 and Are you ready for more?


19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $g(x)+\frac{r(x)}{b(x)}$	end behavior
$g(x)=\frac{x}{x+1}$				
$h(x)=\frac{3x+2}{x+1}$				
$f(x)=\frac{2x^2+x+1}{x+1}$				
$k(x)=\frac{2x^2+x^3+x+1}{x+1}$				
$m(x)=\frac{x+1}{x^2+x^3+x+1}$				

1. Complete the table to explore the end behavior for rational functions.

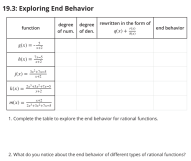
2. What do you notice about the end behavior of different types of rational functions?

SE 2.19 task · PDF p. 2, Activity 19.3

Conceptual understanding

Polynomial division rewrites a rational function as a quotient plus a proper remainder term. Students use that form to explain end behavior, including cases that approach a nonhorizontal line or another polynomial.

SE 2.19 task · PDF p. 2, Activity19.3 #1–2 and Are you ready for more?


19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $g(x)+\frac{r(x)}{b(x)}$	end behavior
$g(x)=\frac{x}{x+1}$				
$h(x)=\frac{3x+2}{x+1}$				
$f(x)=\frac{2x^2+x+1}{x+1}$				
$k(x)=\frac{2x^2+x^3+x+1}{x+1}$				
$m(x)=\frac{x+1}{x^2+x^3+x+1}$				

1. Complete the table to explore the end behavior for rational functions.

2. What do you notice about the end behavior of different types of rational functions?

SE 2.19 task · PDF p. 2, Activity 19.3

Procedural skill and practice

Practice #1 interprets end behavior from an already rewritten rational function. Students must identify the part whose contribution becomes small and the part that governs the eventual output.

SE 2.19 practice · PDF p. 1, Practice #1

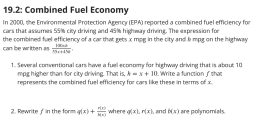

1. The function $f(x)=\frac{5x+1}{x+1}$ can be rewritten in the form $f(x)=5+\frac{17}{x+1}$. What is the end behavior of $y=f(x)$?

SE 2.19 practice · PDF p. 1, #1

Contextual application

Fuel consumption gives meaning to the rational expression and its units. Students must distinguish the behavior of the expression from conclusions warranted by the supplied driving assumptions.

SE 2.19 task · PDF p. 1, Activity 19.2


19.2: Combined Fuel Economy

In 2000, the Environmental Protection Agency (EPA) reported a combined fuel efficiency for cars that assumes 55% city driving and 45% highway driving. The expression for the combined fuel efficiency of a car that gets x mpg in the city and h mpg on the highway can be written as $\frac{100xh}{55h+45x}$.

1. Several conventional cars have a fuel economy for highway driving that is about 10 mpg higher than for city driving. That is, $h=x+10$. Write a function f that represents the combined fuel efficiency for cars like these in terms of x.

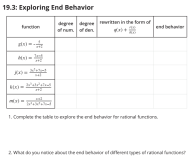
2. Rewrite f in the form $g(x)+\frac{r(x)}{b(x)}$ where $g(x)$, $r(x)$, and $b(x)$ are polynomials.

SE 2.19 task · PDF p. 1, Activity19.2 #1–2

Coherence: horizontal alignment

Polynomial division returns as a way to rewrite rational functions into quotient and remainder parts. That algebraic decomposition explains long-run graph behavior instead of serving only as a division exercise.

SE 2.19 task · PDF p. 2, Activity19.3 #1–2 and Are you ready for more?; 2.13 task · PDF p. 1, Activity 13.2


19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $g(x)+\frac{r(x)}{b(x)}$	end behavior
$g(x)=\frac{x}{x+1}$				
$h(x)=\frac{3x+2}{x+1}$				
$f(x)=\frac{2x^2+x+1}{x+1}$				
$k(x)=\frac{2x^2+x^3+x+1}{x+1}$				
$m(x)=\frac{x+1}{x^2+x^3+x+1}$				

1. Complete the table to explore the end behavior for rational functions.

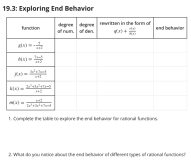
2. What do you notice about the end behavior of different types of rational functions?

SE 2.19 task · PDF p. 2, Activity 19.3

Depth of reasoning

The comparison table asks students to relate numerator and denominator degrees, equivalent form and end behavior. A conclusion needs the proper-remainder relationship, not only a rule memorized from one graph.

SE 2.19 task · PDF p. 2, Activity19.3 #1–2 and Are you ready for more?; TE Student task, anticipated reasoning and Activity Synthesis


19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $g(x)+\frac{r(x)}{b(x)}$	end behavior
$g(x)=\frac{x}{x+1}$				
$h(x)=\frac{3x+2}{x+1}$				
$f(x)=\frac{2x^2+x+1}{x+1}$				
$k(x)=\frac{2x^2+x^3+x+1}{x+1}$				
$m(x)=\frac{x+1}{x^2+x^3+x+1}$				

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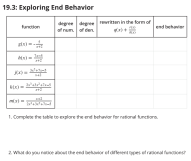
2. What do you notice about the end behavior of different types of rational functions?

SE 2.19 task · PDF p. 2, Activity 19.3

Teacher supports

The teacher gives groups different functions, collects predictions and compares the resulting cases. Supports connect the division representation with the equivalent rational form and provide rehearsal time for the explanation.

TE 19.3 Launch and Activity Synthesis; SE 2.19 task · PDF p. 2, Activity19.3 #1–2 and Are you ready for more?


19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $g(x)+\frac{r(x)}{b(x)}$	end behavior
$g(x)=\frac{x}{x+1}$				
$h(x)=\frac{3x+2}{x+1}$				
$f(x)=\frac{2x^2+x+1}{x+1}$				
$k(x)=\frac{2x^2+x^3+x+1}{x+1}$				
$m(x)=\frac{x+1}{x^2+x^3+x+1}$				

1. Complete the table to explore the end behavior for rational functions.

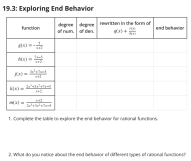
2. What do you notice about the end behavior of different types of rational functions?

SE 2.19 task · PDF p. 2, Activity 19.3

Mathematical practices

MP7 is supported by using quotient and remainder structure to determine eventual behavior. A rational function need not have a horizontal asymptote, so the structure must be checked in each case.

SE 2.19 task · PDF p. 2, Activity19.3 #1–2 and Are you ready for more?; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $g(x)+\frac{r(x)}{b(x)}$	end behavior
$g(x)=\frac{x}{x+1}$				
$h(x)=\frac{3x+2}{x+1}$				
$f(x)=\frac{2x^2+x+1}{x+1}$				
$k(x)=\frac{2x^2+x^3+x+1}{x+1}$				
$m(x)=\frac{x+1}{x^2+x^3+x+1}$				

1. Complete the table to explore the end behavior for rational functions.

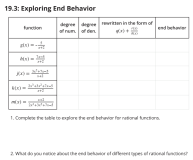
2. What do you notice about the end behavior of different types of rational functions?

SE 2.19 task · PDF p. 2, Activity 19.3

Formative assessment

When a remainder still contains a variable, the teacher suggests testing large inputs to examine whether it approaches zero. This probes the degree comparison that makes the rewritten form informative.

SE 2.19 task · PDF p. 2, Activity19.3 #1–2 and Are you ready for more?; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $g(x) + \frac{r(x)}{q(x)}$	end behavior
$g(x) = \frac{-x}{x+1}$				
$h(x) = \frac{3x+2}{x+1}$				
$j(x) = \frac{2x^2+3x+1}{x+1}$				
$k(x) = \frac{2x^2+3x^2+3x+1}{x+1}$				
$m(x) = \frac{x^3+x^2+1}{x^2+x^2+x+1}$				

1. Complete the table to explore the end behavior for rational functions.

2. What do you notice about the end behavior of different types of rational functions?

SE 2.19 task · PDF p. 2, Activity 19.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

2.20 Rational Equations (Part 1)

Explore lesson 2.20

Standards connection

Students write and solve a batting-average equation, then revise it after three further at-bats without a hit and explain each step. Both numerator and denominator change when subsequent hits are added. HSA.CED.A.1/HSA.REI.A.1: formulate, solve, interpret and revise model explicitly; meaningful model revision, not an assumed feature of contextual wording.

SE 2.20 task · PDF p. 2, Activity20.3 #1–3


20.3: Batting Averages

Tyler is on a school baseball team and he has had 24 base hits out of 110 at bats this year.

1. What is his current batting average?

2. He wants to raise his batting average to .300. How many of the next consecutive at bats need to be base hits to raise his batting average to .300? Write and solve an equation to describe this situation using x for the number of consecutive base hits. Be prepared to explain how you wrote your equation and each of your solving steps.

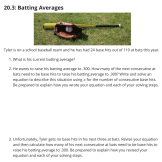
3. Unfortunately, Tyler gets no base hits in his next three at bats. Revise your equation and then calculate how many of his next consecutive at bats need to be base hits to raise his batting average to .300. Be prepared to explain how you revised your equation and each of your solving steps.

SE 2.20 task · PDF p. 2, Activity 20.3

Conceptual understanding

Average-cost and batting-average situations connect a changing numerator with a changing denominator. The batting task explicitly revises the model after three more at-bats without a hit.

SE 2.20 task · PDF p. 2, Activity20.3 #1–3


20.3: Batting Averages

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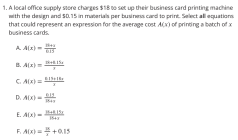
3. Unfortunately, Tyler gets no base hits in his next three at bats. Revise your equation and then calculate how many of his next consecutive at bats need to be base hits to raise his batting average to .300. Be prepared to explain how you revised your equation and each of your solving steps.

SE 2.20 task · PDF p. 2, Activity 20.3

Procedural skill and practice

Practice #1 selects equivalent average-cost equations from fixed setup and per-item costs. A valid expression must divide the complete total cost by the number of items.

SE 2.20 practice · PDF p. 1, Practice #1


1. A local office supply store charges $18 to set up their business card printing machine with the design and $0.15 in materials per business card to print. Select all equations that could represent an expression for the average cost $A(x)$ of printing a batch of x business cards.

A. $A(x) = \frac{18+x}{0.15}$

B. $A(x) = \frac{18+0.15x}{x}$

C. $A(x) = \frac{0.15+18x}{x}$

D. $A(x) = \frac{0.15}{18+x}$

E. $A(x) = \frac{18+0.15x}{18+x}$

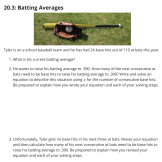
F. $A(x) = \frac{18}{x} + 0.15$

SE 2.20 practice · PDF p. 1, #1

Contextual application

Students determine how many additional at-bats are needed to reach a target batting average. The solution must count whole opportunities and be checked against the assumed future hits; the teacher also discusses revising the supplied model.

SE 2.20 task · PDF p. 2, Activity 20.3


20.3: Batting Averages

Tyler is on a school baseball team and he has had 24 base hits out of 110 at bats this year.

1. What is his current batting average?

2. He wants to raise his batting average to .300. How many of the next consecutive at bats need to be base hits to raise his batting average to .300? Write and solve an equation to describe this situation using x for the number of consecutive base hits. Be prepared to explain how you wrote your equation and each of your solving steps.

3. Unfortunately, Tyler gets no base hits in his next three at bats. Revise your equation and then calculate how many of his next consecutive at bats need to be base hits to raise his batting average to .300. Be prepared to explain how you revised your equation and each of your solving steps.

SE 2.20 task · PDF p. 2, Activity 20.3

Coherence: horizontal alignment

Average-cost work previously interpreted a rational function and its end behavior. Solving a target-value equation now asks which input produces a specified output, with the original domain still in force.

SE 2.20 task · PDF p. 2, Activity20.3 #1–3; 2.18 task · PDF p. 2, Activity 18.2


20.3: Batting Averages

Tyler is on a school baseball team and he has had 24 base hits out of 110 at bats this year.

1. What is his current batting average?

2. He wants to raise his batting average to .300. How many of the next consecutive at bats need to be base hits to raise his batting average to .300? Write and solve an equation to describe this situation using x for the number of consecutive base hits. Be prepared to explain how you wrote your equation and each of your solving steps.

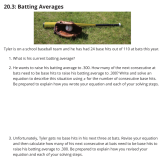
3. Unfortunately, Tyler gets no base hits in his next three at bats. Revise your equation and then calculate how many of his next consecutive at bats need to be base hits to raise his batting average to .300. Be prepared to explain how you revised your equation and each of your solving steps.

SE 2.20 task · PDF p. 2, Activity 20.3

Depth of reasoning

Students explain how they formed and solved the batting equation, then revise it after changed conditions. Adding later hits changes both the hit total and the at-bat total, so a fixed-denominator shortcut is invalid.

SE 2.20 task · PDF p. 2, Activity20.3 #1–3; TE Student task, anticipated reasoning and Activity Synthesis


20.3: Batting Averages

Tyler is on a school baseball team and he has had 24 base hits out of 110 at bats this year.

1. What is his current batting average?

2. He wants to raise his batting average to .300. How many of the next consecutive at bats need to be base hits to raise his batting average to .300? Write and solve an equation to describe this situation using x for the number of consecutive base hits. Be prepared to explain how you wrote your equation and each of your solving steps.

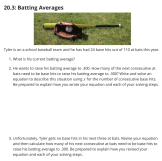
3. Unfortunately, Tyler gets no base hits in his next three at bats. Revise your equation and then calculate how many of his next consecutive at bats need to be base hits to raise his batting average to .300. Be prepared to explain how you revised your equation and each of your solving steps.

SE 2.20 task · PDF p. 2, Activity 20.3

Teacher supports

Teacher guidance starts with numerical average-cost cases if learners cannot write a general expression. The batting launch explains the statistic's meaning before leaving the quantity relationship and equation solving to students.

TE 20.2 Anticipated Misconceptions; 20.3 Launch; SE 2.20 task · PDF p. 2, Activity20.3 #1–3

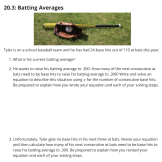

20.3: Batting Averages

SE 2.20 task · PDF p. 2, Activity 20.3

Mathematical practices

MP4 is supported by formulating, solving and revising the batting model in response to a changed situation. The task requires more than evaluating a supplied expression.

SE 2.20 task · PDF p. 2, Activity20.3 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

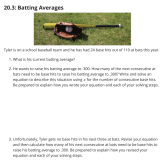

20.3: Batting Averages

SE 2.20 task · PDF p. 2, Activity 20.3

Formative assessment

The teacher compares the equations and solution steps before and after the missed at-bats. The comparison can reveal whether students changed the correct starting totals and interpreted the resulting number of hits.

SE 2.20 task · PDF p. 2, Activity20.3 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


20.3: Batting Averages

SE 2.20 task · PDF p. 2, Activity 20.3

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

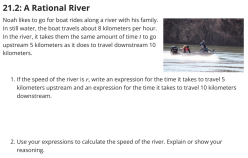
2.21 Rational Equations (Part 2)

Explore lesson 2.21

Standards connection

Students create upstream/downstream travel-time expressions from distances and speed, set equal times and solve river speed with explanation.
HSA.CED.A.1/HSA.REI.A.1/A.2: rational equation from quantities with feasible speed/domain restrictions; context affects the model.

SE 2.21 task · PDF p. 1, Activity21.2 #1–2

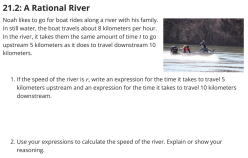

21.2: A Rational River

SE 2.21 task · PDF p. 1, Activity 21.2

Conceptual understanding

The river problem uses equal travel times with different upstream and downstream speeds. The resistance problem then combines reciprocals, extending rational equations beyond a single expression on one side.

SE 2.21 task · PDF p. 1, Activity21.2 #1–2


21.2: A Rational River

SE 2.21 task · PDF p. 1, Activity 21.2

Procedural skill and practice

Practice #1 solves a rational equation by producing and checking candidate values. Denominator restrictions remain relevant after multiplying to obtain a polynomial equation.

SE 2.21 practice · PDF p. 1, Practice #1

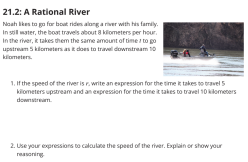
1. Solve $x-1=\frac{x^2-4x+3}{x+2}$ for x.

SE 2.21 practice · PDF p. 1, #1

Contextual application

The river problem equates travel times using rates that change with the current. Students must keep the direction and positive effective speeds consistent when forming and solving the rational equation.

SE 2.21 task · PDF p. 1, Activity 21.2

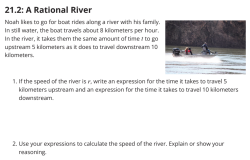

21.2: A Rational River

SE 2.21 task · PDF p. 1, Activity 21.2

Coherence: horizontal alignment

The preceding rational equations develop transformations for a target value. The river task now equates times with different denominators, so a common denominator must preserve the meaning and admissibility of each rate.

SE 2.21 task · PDF p. 1, Activity21.2 #1–2; 2.20 task · PDF p. 1, Activity 20.2


21.2: A Rational River

Noah likes to go for boat rides along a river with his family. In still water, the boat travels about 8 kilometers per hour. In the river, it takes them the same amount of time t to go upstream 5 kilometers as it does to travel downstream 10 kilometers.

1. If the speed of the river is r, write an expression for the time it takes to travel 5 kilometers upstream and an expression for the time it takes to travel 10 kilometers downstream.

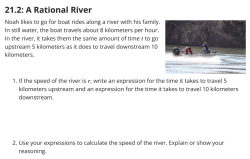
2. Use your expressions to calculate the speed of the river. Explain or show your reasoning.

SE 2.21 task · PDF p. 1, Activity 21.2

Depth of reasoning

Students must justify which speed is added to or subtracted from the boat's still-water speed and why the two time expressions are equal. A calculated value also needs to permit the stated upstream trip.

SE 2.21 task · PDF p. 1, Activity21.2 #1–2; TE Student task, anticipated reasoning and Activity Synthesis


21.2: A Rational River

Noah likes to go for boat rides along a river with his family. In still water, the boat travels about 8 kilometers per hour. In the river, it takes them the same amount of time t to go upstream 5 kilometers as it does to travel downstream 10 kilometers.

1. If the speed of the river is r, write an expression for the time it takes to travel 5 kilometers upstream and an expression for the time it takes to travel 10 kilometers downstream.

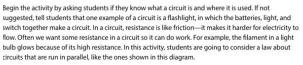
2. Use your expressions to calculate the speed of the river. Explain or show your reasoning.

SE 2.21 task · PDF p. 1, Activity 21.2

Teacher supports

The resistance launch asks learners what circuits are and supplies a familiar flashlight example when needed. This clarifies an unfamiliar context while leaving the reciprocal-resistance relationship to be interpreted and used.

TE 21.3 Launch; SE 2.21 task · PDF p. 1, Activity21.2 #1–2

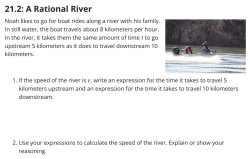

Begin the activity by asking students if they know what a circuit is and where it is used. If not suggested, tell students that one example of a circuit is a flashlight, in which the batteries, light, and switch together make a circuit. In a circuit, resistance is like friction—it makes it harder for electricity to flow. Often we want some resistance in a circuit so it can do work. For example, the filament in a light bulb glows because of its high resistance. In this activity, students are going to consider a law about circuits that are run in parallel, like the ones shown in this diagram.

TE 21.3 Launch

Mathematical practices

MP2 is supported by translating distances and rates into time expressions and interpreting the river speed. The physical law in the resistance task is supplied rather than derived by students.

SE 2.21 task · PDF p. 1, Activity21.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


21.2: A Rational River

Noah likes to go for boat rides along a river with his family. In still water, the boat travels about 8 kilometers per hour. In the river, it takes them the same amount of time t to go upstream 5 kilometers as it does to travel downstream 10 kilometers.

1. If the speed of the river is r, write an expression for the time it takes to travel 5 kilometers upstream and an expression for the time it takes to travel 10 kilometers downstream.

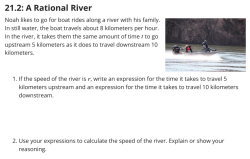
2. Use your expressions to calculate the speed of the river. Explain or show your reasoning.

SE 2.21 task · PDF p. 1, Activity 21.2

Formative assessment

The teacher checks partner agreement on the two river-time expressions before students solve. That checkpoint can separate a context-modeling error from a later algebraic error.

SE 2.21 task · PDF p. 1, Activity21.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


21.2: A Rational River

Noah likes to go for boat rides along a river with his family. In still water, the boat travels about 8 kilometers per hour. In the river, it takes them the same amount of time t to go upstream 5 kilometers as it does to travel downstream 10 kilometers.

1. If the speed of the river is r, write an expression for the time it takes to travel 5 kilometers upstream and an expression for the time it takes to travel 10 kilometers downstream.

2. Use your expressions to calculate the speed of the river. Explain or show your reasoning.

SE 2.21 task · PDF p. 1, Activity 21.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

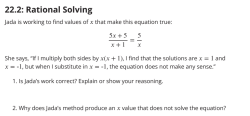
2.22 Solving Rational Equations

Explore lesson 2.22

Standards connection

Students first rule out impossible inputs, then rank equations by anticipated difficulty and choose equations from easy and hard lists to solve. HSA.REI.A.2: denominator restrictions and strategic rational solving; accompanying22.2 requires explanation of an extraneous root, but radical-equation branch not covered here.

SE 2.22 task · PDF p. 1, Activity 22.2 #1–2


22.2: Rational Solving

Jada is working to find values of x that make this equation true:

$$\frac{5x+5}{x+1} = \frac{5}{x}$$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = -1$, but when I substitute in $x = -1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

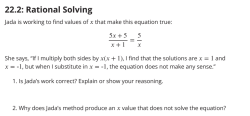
2. Why does Jada's method produce an x value that does not solve the equation?

SE 2.22 task · PDF p. 1, Activity 22.2

Conceptual understanding

The lesson distinguishes an original rational equation from a transformed equation that may admit an excluded input. Students explain how an apparently valid algebraic candidate fails in the original expression.

SE 2.22 task · PDF p. 1, Activity 22.2 #1–2


22.2: Rational Solving

Jada is working to find values of x that make this equation true:

$$\frac{5x+5}{x+1} = \frac{5}{x}$$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = -1$, but when I substitute in $x = -1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

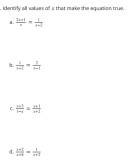
2. Why does Jada's method produce an x value that does not solve the equation?

SE 2.22 task · PDF p. 1, Activity 22.2

Procedural skill and practice

Practice #1 finds all values that satisfy several rational equations. Candidate solutions must be tested against the original denominators, not only the final polynomial equation.

SE 2.22 practice · PDF p. 1, Practice #1


1. Identify all values of x that make the equation true.

a. $\frac{5x+1}{x+2} = \frac{1}{x+2}$

b. $\frac{x}{x+3} = \frac{-3}{x+3}$

c. $\frac{x+1}{x-1} = \frac{x+1}{x+2}$

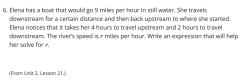
d. $\frac{x+2}{x+3} = \frac{-1}{x+3}$

SE 2.22 practice · PDF p. 1, #1

Contextual application

The river expressions preserve restrictions imposed by effective travel speeds. An algebraic candidate excluded by the original denominator cannot be restored by a transformed equation.

SE 2.22 practice · PDF p. 3, Practice #6


6. Elena has a boat that would go 9 miles per hour in still water. She travels downstream for a certain distance and then back upstream to where she started. Elena notices that it takes her 4 hours to travel upstream and 2 hours to travel downstream. The river's speed is r miles per hour. Write an expression that will help her solve for r.

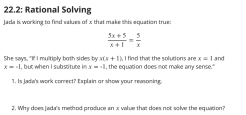
(From Unit 2, Lesson 21.)

SE 2.22 practice · PDF p. 3, Practice #6

Coherence: horizontal alignment

Clearing denominators in Lesson 21 produces a simpler equation. This lesson investigates why the simpler equation may admit an input excluded by the original, so checking is a logical part of solving.

SE 2.22 task · PDF p. 1, Activity 22.2 #1–2; 2.21 task · PDF p. 4, Lesson Summary


22.2: Rational Solving

Jada is working to find values of x that make this equation true:

$$\frac{5x+5}{x+1} = \frac{5}{x}$$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = -1$, but when I substitute in $x = -1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

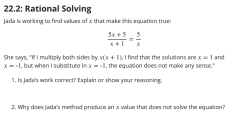
2. Why does Jada's method produce an x value that does not solve the equation?

SE 2.22 task · PDF p. 1, Activity 22.2

Depth of reasoning

Students evaluate Jada's work and explain why her method produces an invalid candidate. Later they compare equations' anticipated difficulty and choose examples to solve, requiring reasons for the chosen approach.

SE 2.22 task · PDF p. 1, Activity 22.2 #1–2; TE Student task, anticipated reasoning and Activity Synthesis


22.2: Rational Solving

Jada is working to find values of x that make this equation true:

$$\frac{5x+5}{x+1} = \frac{5}{x}$$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = -1$, but when I substitute in $x = -1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

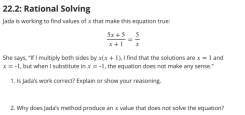
2. Why does Jada's method produce an x value that does not solve the equation?

SE 2.22 task · PDF p. 1, Activity 22.2

Teacher supports

The teacher uses explanation and revision for the extraneous-root discussion and reminds students of quadratic-solving options when needed. Chunking the choice task reduces organizational load while retaining both easier and harder cases.

TE 22.2 Launch and Activity Synthesis; 22.3 Launch; SE 2.22 task · PDF p. 1, Activity 22.2 #1–2


22.2: Rational Solving

Jada is working to find values of x that make this equation true:

$$\frac{5x+5}{x+1} = \frac{5}{x}$$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = -1$, but when I substitute in $x = -1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

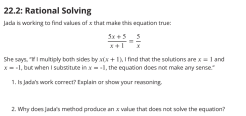
2. Why does Jada's method produce an x value that does not solve the equation?

SE 2.22 task · PDF p. 1, Activity 22.2

Mathematical practices

MP3 is supported by evaluating a proposed solution method and explaining the source of the extraneous value. Checking the original equation provides evidence for accepting or rejecting the candidate.

SE 2.22 task · PDF p. 1, Activity 22.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


22.2: Rational Solving

Jada is working to find values of x that make this equation true:

$$\frac{5x+5}{x+1} = \frac{5}{x}$$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = -1$, but when I substitute in $x = -1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

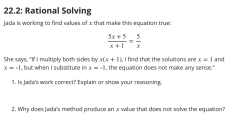
2. Why does Jada's method produce an x value that does not solve the equation?

SE 2.22 task · PDF p. 1, Activity 22.2

Formative assessment

The teacher compares students' reasons for exclusions and their different difficulty rankings. An explanation that merely says an answer failed substitution is pressed toward why the transformation changed the solution set.

SE 2.22 task · PDF p. 1, Activity 22.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


22.2: Rational Solving

Jada is working to find values of x that make this equation true:

$$\frac{5x+5}{x+1} = \frac{5}{x}$$

She says, "If I multiply both sides by $x(x+1)$, I find that the solutions are $x = 1$ and $x = -1$, but when I substitute in $x = -1$, the equation does not make any sense."

1. Is Jada's work correct? Explain or show your reasoning.

2. Why does Jada's method produce an x value that does not solve the equation?

SE 2.22 task · PDF p. 1, Activity 22.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

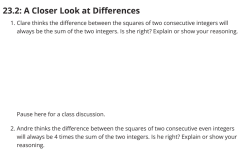
2.23 Polynomial Identities (Part 1)

Explore lesson 2.23

Standards connection

Students prove or refute claims about differences of consecutive integer squares. The even-integer claim is deliberately overstated and requires counterexample or corrected general reasoning. HSA.APR.C.4: use polynomial identity to describe numerical relationships; some true examples do not prove universality.

SE 2.23 task · PDF p. 1, Activity23.2 #1–2


23.2: A Closer Look at Differences

1. Clare thinks the difference between the squares of two consecutive integers will always be the sum of the two integers. Is she right? Explain or show your reasoning.

Pause here for a class discussion.

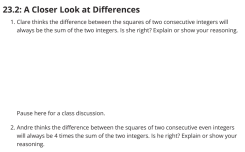
2. Andre thinks the difference between the squares of two consecutive even integers will always be 4 times the sum of the two integers. Is he right? Explain or show your reasoning.

SE 2.23 task · PDF p. 1, Activity 23.2

Conceptual understanding

Numerical differences lead to polynomial identities that describe general relationships. Students distinguish an identity from an equation that happens to be true for selected inputs.

SE 2.23 task · PDF p. 1, Activity23.2 #1–2


23.2: A Closer Look at Differences

1. Clare thinks the difference between the squares of two consecutive integers will always be the sum of the two integers. Is she right? Explain or show your reasoning.

Pause here for a class discussion.

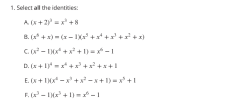
2. Andre thinks the difference between the squares of two consecutive even integers will always be 4 times the sum of the two integers. Is he right? Explain or show your reasoning.

SE 2.23 task · PDF p. 1, Activity 23.2

Procedural skill and practice

Practice #1 selects all identities from several proposed equations. Expanding or factoring must establish equivalence for the relevant values, rather than rely on one successful substitution.

SE 2.23 practice · PDF p. 1, Practice #1


1. Select **all** the identities:

A. $(x+2)^3 = x^3 + 8$

B. $(x^5+x) = (x-1)(x^5+x^4+x^3+x^2+x)$

C. $(x^2-1)(x^4+x^2+1) = x^6-1$

D. $(x+1)^5 = x^5+x^3+x^2+x+1$

E. $(x+1)(x^4-x^3+x^2-x+1) = x^5+1$

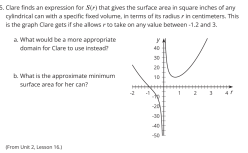
F. $(x^3-1)(x^3+1) = x^6-1$

SE 2.23 practice · PDF p. 1, #1

Contextual application

The cylinder application constrains the permissible radius while comparing surface area. An algebraic relationship must still describe positive dimensions and the fixed-volume situation.

SE 2.23 practice · PDF p. 2, Practice #5


5. Clare finds an expression for $S(r)$ that gives the surface area in square inches of any cylindrical can with a specific fixed volume, in terms of its radius r in centimeters. This is the graph Clare gets if she allows r to take on any value between -1.2 and 3.

a. What would be a more appropriate domain for Clare to use instead?

b. What is the approximate minimum surface area for her can?

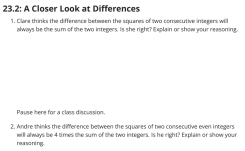
(From Unit 2, Lesson 16.)

SE 2.23 practice · PDF p. 2, Practice #5

Coherence: horizontal alignment

Distributive verification of equivalent forms returns as a general identity question. Numerical agreements can motivate a conjecture, but algebra must establish equality for all permissible inputs.

SE 2.23 task · PDF p. 1, Activity23.2 #1–2; 2.6 task · PDF p. 2, Activity 6.3


23.2: A Closer Look at Differences

1. Clare thinks the difference between the squares of two consecutive integers will always be the sum of the two integers. Is she right? Explain or show your reasoning.

Pause here for a class discussion.

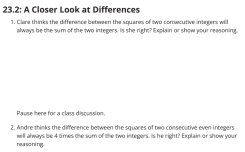
2. Andre thinks the difference between the squares of two consecutive even integers will always be 4 times the sum of the two integers. Is he right? Explain or show your reasoning.

SE 2.23 task · PDF p. 1, Activity 23.2

Depth of reasoning

Students prove or refute claims about differences of squares of consecutive integers and consecutive even integers. The deliberately overstated claim needs a counterexample or corrected general argument, not an assumption that the printed speaker is right.

SE 2.23 task · PDF p. 1, Activity23.2 #1–2; TE Student task, anticipated reasoning and Activity Synthesis


23.2: A Closer Look at Differences

1. Clare thinks the difference between the squares of two consecutive integers will always be the sum of the two integers. Is she right? Explain or show your reasoning.

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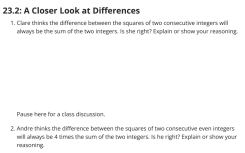
2. Andre thinks the difference between the squares of two consecutive even integers will always be 4 times the sum of the two integers. Is he right? Explain or show your reasoning.

SE 2.23 task · PDF p. 1, Activity 23.2

Teacher supports

Teacher guidance helps students express consecutive even integers and organize distributive work systematically. Concrete cases are compared with the variable expressions when learners find the general notation difficult.

TE 23.2 and 23.3 Anticipated Misconceptions; SE 2.23 task · PDF p. 1, Activity23.2 #1–2


23.2: A Closer Look at Differences

1. Clare thinks the difference between the squares of two consecutive integers will always be the sum of the two integers. Is she right? Explain or show your reasoning.

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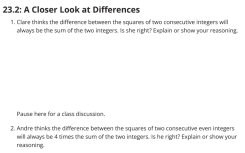
2. Andre thinks the difference between the squares of two consecutive even integers will always be 4 times the sum of the two integers. Is he right? Explain or show your reasoning.

SE 2.23 task · PDF p. 1, Activity 23.2

Mathematical practices

MP3 is supported by testing and justifying universal claims. A collection of examples can motivate an identity but does not prove that it holds for every permitted input.

SE 2.23 task · PDF p. 1, Activity23.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


23.2: A Closer Look at Differences

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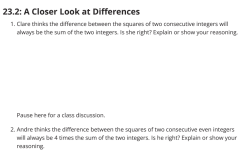
2. Andre thinks the difference between the squares of two consecutive even integers will always be 4 times the sum of the two integers. Is he right? Explain or show your reasoning.

SE 2.23 task · PDF p. 1, Activity 23.2

Formative assessment

The teacher records how students rewrite the numerical differences and carries those methods into the general claim discussion. This reveals whether students can move from checking cases to explaining a universal relationship.

SE 2.23 task · PDF p. 1, Activity23.2 #1–2; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


23.2: A Closer Look at Differences

1. Clare thinks the difference between the squares of two consecutive integers will always be the sum of the two integers. Is she right? Explain or show your reasoning.

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SE 2.23 task · PDF p. 1, Activity 23.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

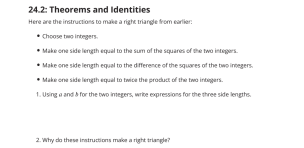
2.24 Polynomial Identities (Part 2)

Explore lesson 2.24

Standards connection

Students express triangle sides using two integers and prove the identity underlying the construction. Original teacher synthesis intentionally asks for failing choices and separates identity truth from geometric restrictions. HSA.APR.C.4: prove identity and use it for numerical relationships; positivity/nondegeneracy constraints belong to triangle use, not identity truth.

SE 2.24 task · PDF p. 2, Activity24.2 #1–2; warm-up instructions PDFp1


24.2: Theorems and Identities
Here are the instructions to make a right triangle from earlier:
- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.

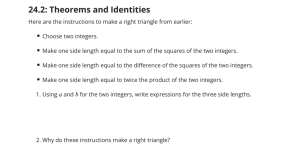
2. Why do these instructions make a right triangle?

SE 2.24 task · PDF p. 2, Activity 24.2

Conceptual understanding

The triangle task separates a polynomial identity from restrictions on using it for physical side lengths. Students prove the algebraic relationship while considering whether the chosen integers actually create a nondegenerate triangle.

SE 2.24 task · PDF p. 2, Activity24.2 #1–2; warm-up instructions PDFp1


24.2: Theorems and Identities
Here are the instructions to make a right triangle from earlier:
- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.

2. Why do these instructions make a right triangle?

SE 2.24 task · PDF p. 2, Activity 24.2

Procedural skill and practice

Practice #1 asks whether a stated factorization is an identity and requires an explanation. Symbolic equivalence, rather than a few numerical matches, establishes the claim.

SE 2.24 practice · PDF p. 1, Practice #1

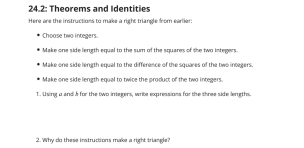

1. Is $a^5 + b^5 = (a^2 + b^2)(a^3 - a^2b^2 + b^3)$ an identity? Explain or show your reasoning.

SE 2.24 practice · PDF p. 1, #1

Coherence: horizontal alignment

The preceding identity definition is tested against triangle constructions. Some proposed triples intentionally fail; students must distinguish a general identity from expressions that meet both the equation and positive-length constraints.

SE 2.24 task · PDF p. 2, Activity24.2 #1–2; warm-up instructions PDFp1; 2.23 task · PDF p. 3, Lesson Summary


24.2: Theorems and Identities
Here are the instructions to make a right triangle from earlier:
- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.

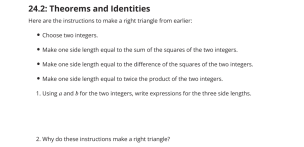
2. Why do these instructions make a right triangle?

SE 2.24 task · PDF p. 2, Activity 24.2

Depth of reasoning

Students explain why the side expressions satisfy the Pythagorean relationship and identify failing choices. These failures are intentional: teacher synthesis explicitly requests a construction that does not work, so they are not treated as a curriculum error.

SE 2.24 task · PDF p. 2, Activity24.2 #1–2; warm-up instructions PDFp1; TE Student task, anticipated reasoning and Activity Synthesis


24.2: Theorems and Identities
Here are the instructions to make a right triangle from earlier:
- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.

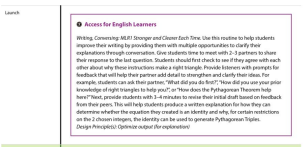
2. Why do these instructions make a right triangle?

SE 2.24 task · PDF p. 2, Activity 24.2

Teacher supports

The teacher uses several partners, feedback questions and written revision to strengthen the identity explanation. Guidance also distinguishes the hypotenuse and geometric side restrictions from the identity's general truth.

TE 24.2 Launch; SE 2.24 task · PDF p. 2, Activity24.2 #1–2; warm-up instructions PDFp1


Launch

Access for English Learners

Writing, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners to share their response to the last question. Students should first check to see if they agree with each other about why these instructions make a right triangle. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, "What did you do first?", "How did you use your prior knowledge of right triangles to help you?", or "How does the Pythagorean Theorem help here?" Next, provide students with 3–4 minutes to revise their initial draft based on feedback from their peers. This will help students produce a written explanation for how they can determine whether the equation they created is an identity and why, for certain restrictions on the 2 chosen integers, the identity can be used to generate Pythagorean Triples.

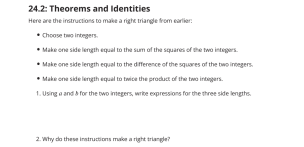
Design Principle(s): Optimize output (for explanation)

TE 24.2 Launch

Mathematical practices

MP3 is supported by proving equivalence and evaluating the restrictions behind the construction claim. Numerical examples alone do not settle either the general identity or the geometry of every integer choice.

SE 2.24 task · PDF p. 2, Activity24.2 #1–2; warm-up instructions PDFp1; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


24.2: Theorems and Identities
Here are the instructions to make a right triangle from earlier:
- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.

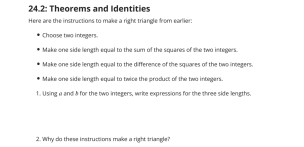
2. Why do these instructions make a right triangle?

SE 2.24 task · PDF p. 2, Activity 24.2

Formative assessment

The teacher records successful and failing triangle choices, then asks students to justify the identity with different algebraic approaches. This can expose confusion between a true equation and a valid geometric construction.

SE 2.24 task · PDF p. 2, Activity24.2 #1–2; warm-up instructions PDFp1; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


24.2: Theorems and Identities
Here are the instructions to make a right triangle from earlier:
- Choose two integers.
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- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.

2. Why do these instructions make a right triangle?

SE 2.24 task · PDF p. 2, Activity 24.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

Unassessed: Contextual application. The selected evidence does not establish contextual application in this lesson.

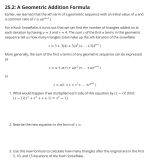
2.25 Summing Up

Explore lesson 2.25

Standards connection

Students use an earlier polynomial identity and guided multiplication to derive the finite geometric-sum formula, then apply it to snowflake totals. HSA.SSE.B.4/HSA.SSE.A.2: derivation plus application, with common ratio unequal to one; supplied hint limits independent strategy selection.

SE 2.25 task · PDF p. 2, Activity25.2 #1–3

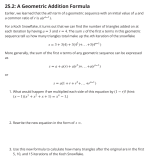

25.2: A Geometric Addition Formula

SE 2.25 task · PDF p. 2, Activity 25.2

Conceptual understanding

The snowflake setting connects a long geometric sum with a shorter expression using an earlier polynomial identity. Students derive the finite-sum formula and then interpret accumulation in the antibiotic situation.

SE 2.25 task · PDF p. 2, Activity25.2 #1–3

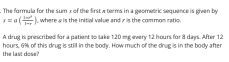

25.2: A Geometric Addition Formula

SE 2.25 task · PDF p. 2, Activity 25.2

Procedural skill and practice

Practice #1 calculates the amount of a repeated medicine dose remaining after the final dose. The number of doses and retained fraction must be identified before applying the supplied finite-sum formula.

SE 2.25 practice · PDF p. 1, Practice #1


1. The formula for the sum s of the first n terms in a geometric sequence is given by $s = a\left(\frac{1-r^n}{1-r}\right)$, where a is the initial value and r is the common ratio.

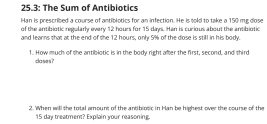
A drug is prescribed for a patient to take 120 mg every 12 hours for 8 days. After 12 hours, 6% of this drug is still in the body. How much of the drug is in the body after the last dose?

SE 2.25 practice · PDF p. 1, #1

Contextual application

The antibiotic task accumulates surviving amounts from repeated doses. Choosing the ratio and term count requires distinguishing an individual dose from the total present at the specified time.

SE 2.25 task · PDF p. 3, Activity 25.3


25.3: The Sum of Antibiotics

Han is prescribed a course of antibiotics for an infection. He is told to take a 150 mg dose of the antibiotic regularly every 12 hours for 15 days. Han is curious about the antibiotic and learns that at the end of the 12 hours, only 5% of the dose is still in his body.

1. How much of the antibiotic is in the body right after the first, second, and third doses?

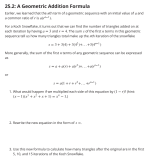
2. When will the total amount of the antibiotic in Han be highest over the course of the 15 day treatment? Explain your reasoning.

SE 2.25 task · PDF p. 3, Activity25.3 #1–2

Coherence: horizontal alignment

Earlier accumulation of geometric terms motivates a general sum. Multiplying and subtracting two aligned sums now uses an identity to justify a shortcut rather than relying on repeated addition.

SE 2.25 task · PDF p. 2, Activity25.2 #1–3; 1.11 task · PDF p. 2, Activity 11.3

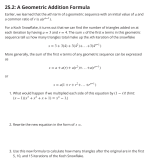

25.2: A Geometric Addition Formula

SE 2.25 task · PDF p. 2, Activity 25.2

Depth of reasoning

Students explain the cancellation after multiplying a finite sum and use it to isolate the total. The derivation depends on a common ratio different from one; the suggested multiplier is supplied, so strategy selection is scaffolded.

SE 2.25 task · PDF p. 2, Activity25.2 #1–3; TE Student task, anticipated reasoning and Activity Synthesis


25.2: A Geometric Addition Formula

SE 2.25 task · PDF p. 2, Activity 25.2

Teacher supports

The teacher uses a simpler identity when the sign of the multiplier is confusing and pictures successive retained doses when the percentage is misunderstood. These supports connect algebraic cancellation and accumulation without replacing the general reasoning.

TE 25.2 and 25.3 Anticipated Misconceptions; SE 2.25 task · PDF p. 2, Activity25.2 #1–3


25.2: A Geometric Addition Formula

SE 2.25 task · PDF p. 2, Activity 25.2

Mathematical practices

MP7 is supported by recognizing the cancellation structure that turns a long sum into a useful formula. Repeating additions alone would not establish this structural connection.

SE 2.25 task · PDF p. 2, Activity25.2 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis

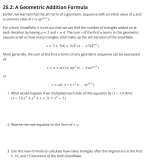


SE 2.25 task · PDF p. 2, Activity 25.2

Formative assessment

The teacher asks how students identified the initial term, ratio and number of terms in each context. A mistaken retention factor or timing interpretation can then be distinguished from a formula-substitution error.

SE 2.25 task · PDF p. 2, Activity25.2 #1–3; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis



SE 2.25 task · PDF p. 2, Activity 25.2

Unassessed: Summative assessment. Protected assessment items and scoring guidance were not reviewed.

Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

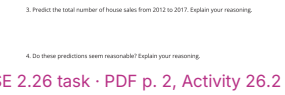
2.26 Using the Sum

Explore lesson 2.26

Standards connection

Students predict accumulated house sales under a stated growth trend and assess reasonableness. The model assumption is supplied but evaluation of the prediction is required. HSA.SSE.B.4/HSA.SSE.A.1: interpret a finite geometric total and judge model output; no empirical validation is required merely to meet this task.

SE 2.26 task · PDF p. 2, Activity26.2 #3–4; setup and#1–2 on PDFp1

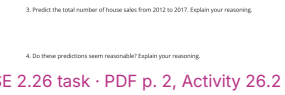


SE 2.26 task · PDF p. 2, Activity 26.2

Conceptual understanding

Students apply the finite-sum formula to housing forecasts and regular investments. They must interpret accumulated totals, reasonableness and deposit timing rather than only substitute into a formula.

SE 2.26 task · PDF p. 2, Activity26.2 #3–4; setup and#1–2 on PDFp1

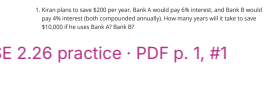


SE 2.26 task · PDF p. 2, Activity 26.2

Procedural skill and practice

Practice #1 compares how long two annual-interest accounts take to reach a saving target. The contribution convention and compounding model must be used consistently when comparing the banks.

SE 2.26 practice · PDF p. 1, Practice #1



SE 2.26 practice · PDF p. 1, #1

Contextual application

Repeated contributions and compound growth require counting the growth periods for each deposit. Students compare their predicted total with the computed sum and distinguish contributions from earned interest.

SE 2.26 task · PDF p. 3, Activity 26.3



SE 2.26 task · PDF p. 3, Activity26.3 #1–3

Coherence: horizontal alignment

The finite-sum derivation is applied to housing and repeated investment contributions. Selecting the first term and number of terms now depends on what is counted and when each change occurs.

SE 2.26 task · PDF p. 2, Activity26.2 #3–4; setup and#1–2 on PDFp1; 2.25 task · PDF p. 2, Activity 25.2



SE 2.26 task · PDF p. 2, Activity 26.2

Depth of reasoning

The housing task asks students to explain the total being predicted and whether the result is reasonable. The saving task compares contributions with earned interest, so the size and meaning of the result both matter.

SE 2.26 task · PDF p. 2, Activity26.2 #3–4; setup and#1–2 on PDFp1; TE Student task, anticipated reasoning and Activity Synthesis

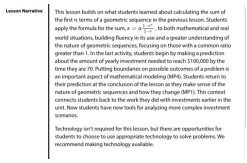


SE 2.26 task · PDF p. 2, Activity 26.2

Teacher supports

The preparation recommends technology be available but does not require it, and the launch clarifies deposits at the start of each year. If students cannot begin, the teacher suggests expanding the first few years before using the general sum.

TE Lesson Narrative; SE 2.26 task · PDF p. 2, Activity26.2 #3–4; setup and#1–2 on PDFp1


Lesson Narrative

This lesson builds on what students learned about calculating the sum of the first n terms of a geometric sequence in the previous lesson. Students apply the formula for the sum, $s = a\frac{1-r^n}{1-r}$, to both mathematical and real world situations, building fluency in its use and a greater understanding of the nature of geometric sequences, focusing on those with a common ratio greater than 1. In the last activity, students begin by making a prediction about the amount of yearly investment needed to reach $100,000 by the time they are 70. Putting boundaries on possible outcomes of a problem is an important aspect of mathematical modeling (MP4). Students return to their prediction at the conclusion of the lesson as they make sense of the nature of geometric sequences and how they change (MP1). This context connects students back to the work they did with investments earlier in the unit. Now students have new tools for analyzing more complex investment scenarios.

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems. We recommend making technology available.

TE Lesson Narrative

Mathematical practices

MP2 is supported by interpreting the terms of a housing or investment sum in their original units and timing. The stated trend or fixed rate is an assumption of the supplied model, not a law established by the calculation.

SE 2.26 task · PDF p. 2, Activity26.2 #3–4; setup and#1–2 on PDFp1; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


3. Predict the total number of house sales from 2012 to 2017. Explain your reasoning.

4. Do these predictions seem reasonable? Explain your reasoning.

SE 2.26 task · PDF p. 2, Activity 26.2

Formative assessment

The teacher compares how students chose the initial term, common ratio and term count and returns to their initial prediction. This can reveal an extra compounding period or a mistaken count of deposits.

SE 2.26 task · PDF p. 2, Activity26.2 #3–4; setup and#1–2 on PDFp1; TE Activity monitoring, Anticipated Misconceptions, Activity Synthesis and Lesson Synthesis


3. Predict the total number of house sales from 2012 to 2017. Explain your reasoning.

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SE 2.26 task · PDF p. 2, Activity 26.2

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Unassessed: Coherence: vertical alignment. An explicit prior-grade connection was not verified for this lesson.

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